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CAVE: A Computer Code for Two-Dimensional Transient Heating Analysis of Conceptual Thermal Protection Systems for Hypersonic Vehicles

Kenneth A. Rathjen

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NOVEMBER 1977

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Prepared for
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National Aeronautics
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SUMMARY

This report describes a digital computer code CAVE (Conduction Analysis Via Eigenvalues), which finds application in the analysis of two-dimensional transient heating of hypersonic vehicles. The code is an extension of the work reported in NASA CR-2435 for the inverse conduction problem. CAVE is written in FORTRAN IV and is operational on both IBM 360-67 and CDC 6600 computers.

The main advantages of CAVE over more conventional thermal analyzer codes are:

- The method of solution is a hybrid analytical-numerical technique that is inherently stable permitting large time steps even with the best of conductors having the finest of mesh size. This method can provide a factor-of-five reduction in machine time compared to conventional explicit finite difference methods when structures with small time constants are analyzed over long flight trajectories.
- The aerodynamic heating boundary conditions are calculated by the code based on the input flight trajectory (i.e., altitude, velocity and angle of attack as functions of time) rather than calculated external to the code and then entered as input data.
- The code computes the network conduction and convection links, as well as capacitance values, given basic geometrical and mesh sizes, for four geometries (leading edges, cooled panels, X-24C structure and slabs).
- The output from the code at each time interval includes the steady-state solution corresponding to the boundary conditions for that time interval.
- The code also permits direct input of the heat transfer couplings, node capacitances and boundary conditions.

This report is primarily a user's manual for the CAVE code. Input and output formats are presented and explained. Sample problems are included. A brief summary of the hybrid analytical-numerical technique, which utilizes eigenvalues (thermal frequencies) and eigenvectors (thermal mode vectors) is given in an appendix. Other appendixes include the aerodynamic heating equations that have been incorporated in the code and flow charts.

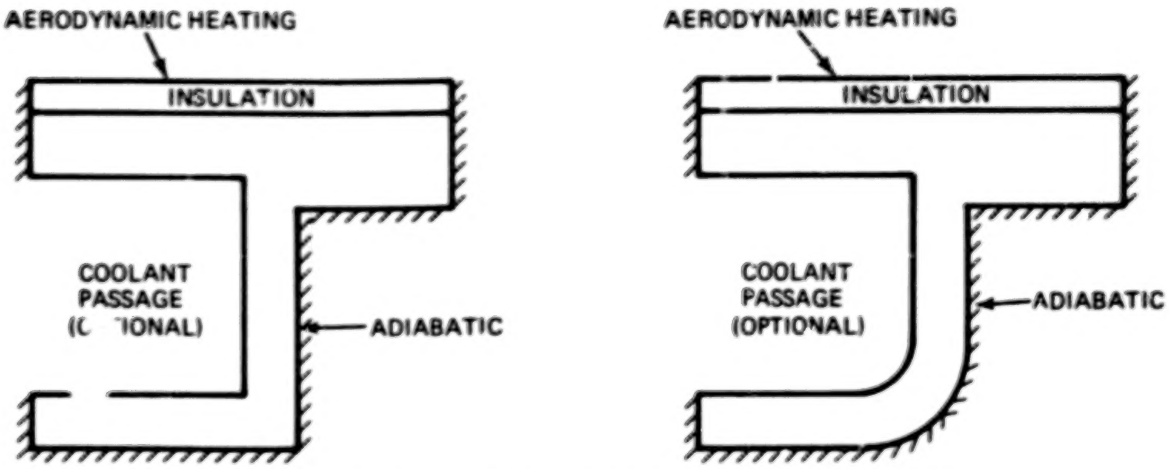
Section 1

INTRODUCTION

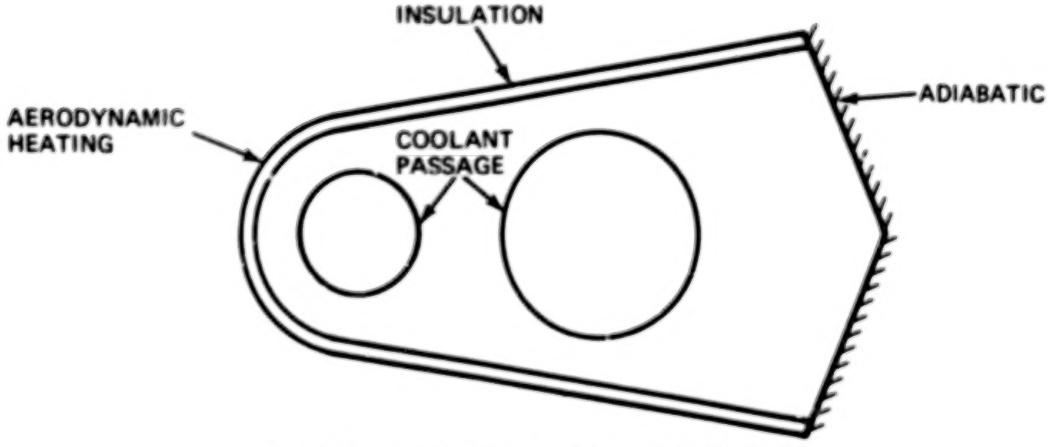
The computer code CAVE (Conductive Analysis Via Eigenvalues) provides a very convenient and efficient tool for predicting the temperatures within thermal protection systems for hypersonic vehicles.

The CAVE code is convenient to use because, first, the boundary conditions (convective heat transfer coefficient and adiabatic wall temperature) are calculated by the code based on the input values for altitude, velocity and angle of attack as functions of time. And, second, because the conduction and convection links between nodes, and the capacitance of each node are calculated by the code for leading edges, slabs and cooled panels (Fig. 1). The code also permits direct input of the heat transfer couplings, node capacitances and boundary conditions for other two-dimensional problems.

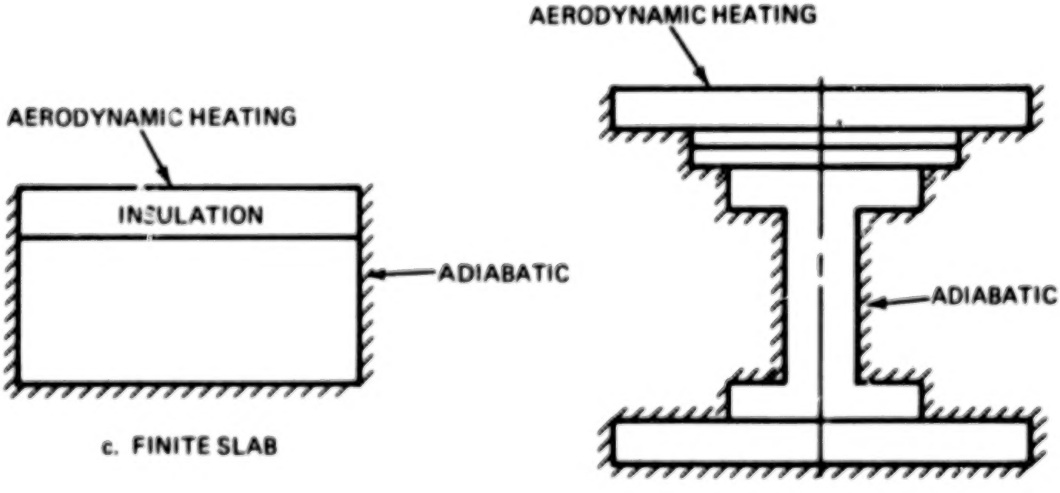
CAVE can be very efficient in the use of computer time because the method employed to solve the partial differential heat conduction equation is a hybrid analytical-numerical (HAN) technique. In this method, spatial derivatives are replaced by appropriate finite difference representations and the temporal derivatives are retained as ordinary derivatives. In effect the problem is subdivided into a number of uniform temperature systems or nodes that are coupled and changing in temperature. The problem is thereby specified by a set of first order, linear, ordinary differential equations. The solution to the set of equations is expressed in terms of eigenvectors (thermal mode vectors for the system) and eigenvalues (thermal frequencies of the system). Appendix A gives details of the method. The important thing to note is that this method is particularly efficient in the use of computer time when the heat flux response is contained in the first few thermal modes (characteristic of materials with high thermal diffusivity) or if the response for a large number of time increments is required, which is precisely the situation in predicting the temperatures throughout the flight trajectory of a hypersonic vehicle. A reduction by



a. COOLED PANEL (SQUARE- AND ROUND-CORNERED PASSAGES)



b. LEADING EDGE WITH COOLANT PASSAGES



c. FINITE SLAB

d. X-24C STRUCTURE

FIG. 1 GEOMETRIES BUILT INTO CAVE CODE

a factor of five in computer time can be expected over conventional explicit finite difference codes for typical flight trajectory analyses. The savings in computer time is due to the HAN method being inherently stable and, therefore, permitting large time steps.

This report is basically a user's manual for CAVE. Section 2 describes the overall operation and running of CAVE while Section 3 discusses the details of the leading edge problem. The input data format is presented and the output from sample problems is reviewed. Section 4 provides a similar treatment for the slab, cooled panel and X-24C geometries. Section 5 discusses an arbitrarily shaped geometry. The appendixes present such information as the deta ls of the HAN method (Appendix A), the aerodynamic heating equations (Appendix B), a discussion of the treatment of radiation (Appendix C), a brief description of the CAVE subroutines (Appendix D), a discussion of nonlinearities and time dependency of h and T_{AW} (Appendix E), and a derivation of the eigenvalue/eigenvector solution (Appendix F).

Mr. James L. Hunt, of the High Speed Aerodynamics Division, Langley Research Center, Virginia, served as the NASA technical monitor for the program.

At Grumman, the contract was administered by the Advanced Development office, under Mr. Fred Berger, Manager of Advanced Development System Engineering. The Study Manager was Dr. Kenneth A. Rathjen.

Mr. Michael J. Rossi served as numerical analysis consultant for the program. Mr. Rossi developed the numerical method and the matrix subroutines package under contract NAS 1-11818, "Lateral Conduction Effects on Heat-Transfer Data Obtained with the Phase-Change Paint Technique," described in report NASA CR-2435 with co-author George Maise. Mr. Hunt also served as technical monitor for that program.

Messrs. William Timlen and Charles Osonitsch were of considerable assistance in providing the appropriate aerodynamic heating functions. Mr. Timlen also gave important support by running Grumman's TTA1 computer code to obtain independent checks on the CAVE code. Mr. Brian Martin developed subroutine X-24C.

The many helpful discussions with Dr. Gianky Da Forno are gratefully acknowledged.

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Section 2

DESCRIPTION OF CAVE CODE OPERATION

This section provides an overview of the CAVE code capabilities, input/output, and method of solution.

CAVE has been designed with the convenience of the user in mind. Usual operation of CAVE requires only the following elemental information from the user:

- Selection of one of the built-in configurations of Figure 1 or the general geometry option
- Specification of the surface emissivity and background radiation temperature for problems involving radiation heat transfer
- Specification of the material density, specific heat and thermal conductivity (the latter two of which can be temperature dependent)
- Geometry-type information such as overall dimensions and grid network sizes
- Initial temperature distribution
- Flight trajectory, i.e., altitude, velocity and angle of attack as functions of time in tabular form. This information is used by the code to predict the aerodynamic heat transfer coefficient, h, and adiabatic wall temperature, T_{AW} as functions of time. Optionally the user may supply tables for h and T_{AW} as functions of time and distance
- Specification of the time step intervals

With the above information specified, the problem solution is accomplished in the following sequence by the code:

1. Storage requirements for the various arrays are determined and allocated.
2. The geometry is discretized into elements and the volumes, conduction areas and lengths are computed. (A unit depth is assumed by the code.) For the general geometry problem these quantities are input data.

Then for each time step the following are done:

3. Using the temperature distribution at the start of the time step, the thermal properties of the materials are determined, followed by the capacitances and conductances for the network. This step is exercised just once if the material properties are independent of temperature.

4. Using the flight trajectory data at the start and end of the time step, the code establishes the time average convective heat transfer coefficient at each surface node and the corresponding time average adiabatic wall temperatures. Appendix B gives the particular aerodynamic heating equations used.

5. The convective heat transfer couplings, due either to aerodynamic heating or internal coolant flow, at each surface node are then determined by taking the product of the convective coefficient and the surface area. These couplings are then modified to account for radiation if it is being considered. Appendix C gives the details of the linearized treatment that is given to the radiation heat transfer.

At this juncture we may visualize the code as being faced with the task of finding the solution to the following system of n first-order linear differential equations with constant coefficients:

$$C_i \frac{dT_i}{dt} = K_{ij} (T_j - T_i) + H_i (T_{AW,\,i} - T_i) \qquad i = 1, n \qquad \text{Eq. (1)}$$

where C_i = thermal capacitance of node i

K_{ij} = conductive coupling between nodes i and j

T_i = temperature of node i

T_j = temperature of node j which is adjacent to node i

H_i = convective coupling between node i and the fluid (for interior nodes $H_i = 0$)

t = time

$T_{AW,i}$ = adiabatic wall temperature of the fluid in contact with node i

There are n such coupled differential equations, one for each of the n nodes.

It may be interesting to digress for a moment to note that the usual thermal analyzers take equation (1) a step further and replace the ordinary derivative $\frac{dT_i}{dt}$ with a finite difference approximation. Depending on the form of the approximation either an explicit or implicit algorithm is obtained. In the common explicit and implicit formulations, the T_i's and T_j's are taken to be constant during the time step interval. In the current HAN method, the ordinary derivative is retained and the T_i's and T_j's are treated as time-dependent variables in Eq. (1). This leads to a more accurate solution with no limitation on the time step from a stability standpoint. However, in solving Eq. (1), the HAN method treats the C_i, K_{ij}, H_i and $T_{AW,i}$ as constants. This is necessary, as discussed in Appendix E, for an eigenvalue solution to exist. The technique used within CAVE to handle variations in these parameters is to subdivide the total time interval (i.e., the flight trajectory) and take these parameters to be piecewise constant within each time subinterval.

Thus, the single problem of determining the temperature distribution in the structure for the entire flight trajectory where the boundary conditions are varying is solved by considering a number of subproblems where the boundary conditions are piecewise constant. These subproblems are interconnected in that the temperature at the end of one time subinterval becomes the initial temperature for the next time subinterval. It should be noted that the time subintervals, or time steps in the HAN method, are typically of the order of seconds or tens of seconds which is probably 100 to 1000 times larger than is permissible with the explicit method.

CAVE arithmetically averages the convective coefficient and adiabatic temperature at the beginning and end of the time interval. Therefore in selecting the time subintervals, the user should be guided by the variation in the flight trajectory with particular concern for abrupt changes that affect the convective heating. Assuming the flight trajectory table has been set up with these important points of change, time subintervals equal to those used in the flight trajectory table will very often prove satisfactory. For those problems in which the temperature dependency of the material properties plays a dominant role for some reason, or if radiation heat transfer is of great importance, a second run with smaller subintervals should be made to determine the effect of subinterval selection on the predicted temperatures.

The system of equations given in Eq. (1) has the following exact solution for a particular time-subinterval (refer to Appendix F):

$$T_i = T_{\infty_i} + \sum_{j=1}^{n} c_{ij} \exp(\lambda_j t) \qquad \text{Eq. (2)}$$

where

T_i = temperature at node i at time t seconds into the time subinterval

T_{∞_i} = steady-state temperature at node i for the particular time subinterval

c_{ij} = constants that depend on the T_{∞_i}, a set of eigenvectors of a matrix A, and the temperatures of the nodes at the start of the time subinterval

λ_j = the eigenvalues of a matrix A

t = time into the particular time subinterval. If τ represents the time in the flight trajectory, and if τ_s and τ_e represent the time at the start and end of a time subinterval, then the following relationships hold:

$$0 \leq t \leq \tau_e - \tau_s \text{ and } \tau = \tau_s + t \text{ for } \tau_s \leq t \leq \tau_e$$

A = symmetric matrix whose elements depend on the C_i, K_{ij} and H_i of Eq. (1). (Refer to Appendixes A and F)

Considering a thermal network with 100 nodes, there are then 100 eigenvalues and eigenvectors to be determined and used in Eq. (2). Considerable machine time can be saved by calculating only those eigenvalues and eigenvectors that are "significant" or "dominant". This was noted very aptly by Maise and Rossi in NASA CR-2435 and used by them in the CAPE code for the inverse heat transfer problem of finding the boundary conditions given the temperature history. When the series in Eq. (2) is truncated to the "dominant" terms, we obtain:

$$T_i = T_{\infty_i} + \sum_{j=1}^{ne} c_{ij} \exp(\lambda_j t) \qquad \text{Eq. (3)}$$

where ne is a number substantially less than n. It represents the number of dominant eigenvalues and eigenvectors that will be found and utilized by the code. This is an input number decided upon by the user. Values of ne from 3 to 5 are recommended for most problems. Appendix A presents a discussion on this subject.

With this background information, we are in a position to continue with the sequence that the CAVE code undergoes in finding the temperature history of the structure throughout the flight trajectory. The next four steps involve matrix subroutines which for the most part were written and developed under contract NAS 1-11818 by M. J. Rossi. The sequence then continues from p. 6 as follows:

6. Set up a matrix A in compact form which depends on the C_i, K_{ij} and H_i of Eq. (1) (Refer to Appendix F).

7. Obtain the ne dominant eigenvectors and eigenvalues of matrix A using Jennings* method of simultaneous vector iteration.

8. Determine the steady-state solution to Eq. (1).

9. Calculate the c_{ij} of Eq. (3) for i = 1, 2, . . . , n and j = 1, 2, . . . , ne.

10. Calculate the temperatures of the nodes at the end of the time subinterval using Eq. (3).

11. Set the initial temperatures for the next time subinterval equal to the final temperature of the present subinterval. Increment the flight trajectory time.

12. Repeat steps 3 through 11 until the final time has been reached. The solution is then completed.

The output from CAVE is for the most part self-explanatory and will be reviewed in detail in the following sections when sample problems are considered. In essence, there are three sections to the output. First, there is a partial feedback of the input data, including: geometric parameters, material properties, flight trajectory, or convective heat-transfer coefficient and adiabatic wall temperature, as the case may be, and initial temperature distribution. Secondly, there are the node numbers, material numbers, capacitances and conductances that were calculated by the code

*A. Jennings, "A Direct Iteration Method of Obtaining Latent Roots and Vectors of a Symmetric Matrix," Proc. Cambridge Phil. Soc., 63, 1967, pp. 755-765.

for each node. And thirdly, there are printed out for each time subinterval the time average heat transfer coefficients, heat transfer couplings and adiabatic wall temperatures. Also printed out are Mach number, altitude, velocity, angle of attack and node temperatures at the end of each subinterval. As an aid to the user for a better feel of the problem being analyzed, the following items are also printed out: the steady-state temperature distribution for the time subinterval and the time-integrated heat input to each boundary node.

Although CAVE has been designed to be most convenient for users interested in predicting structural temperatures of hypersonic vehicles, the code also proves convenient for analyzing the geometries given in Figure 1, subjected to other than the normal aerodynamic heating. The user may take advantage of the automatic division of these geometries by the code and supply the particular boundary conditions of his problem as input data. CAVE also proves a valuable code for analyzing geometries other than those given in Figure 1, i.e., whenever the time constant of the system is small compared to total time of interest. In this case, the HAN method of CAVE offers significant machine-time savings compared to conventional methods. Sections 3 and 4 consider in detail the built-in geometries and contain sample cases. Section 5 discusses the general two-dimensional capabilities of CAVE.

Section 3

LEADING EDGE GEOMETRY

3.1 DISCUSSION

This section presents the leading edge geometry that has been incorporated into CAVE and discusses how this geometry is discretized into nodes by the LEAD4 subroutine.* This section also presents the input data format for this geometry.

Figure 2 shows the leading edge geometry that is tacitly assumed by subroutine LEAD4 when it generates a nodal network. The insulating layer can be eliminated, as can either one or both of the coolant passages. These eliminations are accomplished very simply by using input values of zero for the insulation thickness and coolant passage radii.

Figure 3 shows the grid network for the leading edge. The nose region is divided into elements by concentric arcs and rays. The wedge portion is divided into rectangular elements, except near the coolant passages where odd shapes are encountered, and near the centerline, where the elements are trapezoidal. The calculation of the node capacitances is a straight-forward matter and it is done exactly for all elements including the trapezoids. The calculation of the conductances is also a straight-forward matter except near the coolant passages where the conductances are approximated using an effective area and length between nodes. In regard to the approximation, the code requires that the following relationships be maintained (Fig. 3):

ΔX_1 = RP1, radius of nose coolant passage

$\Delta X_i = \Delta X_{i+1} = \Delta X_{i+2} = \frac{2}{3}$ RP2, where RP2 = radius of aft coolant passage

(and so, for example, i = 4 in Fig. 3)

*LEAD4 is an expanded version of LEAD, which is a subroutine for leading edges written by George Maise under contract NAS 1-11818 and reported in NASA CR-2435 by George Maise and Michael J. Rossi. The expanded version can handle cooling passages and a layer of insulation applied around the leading edge.

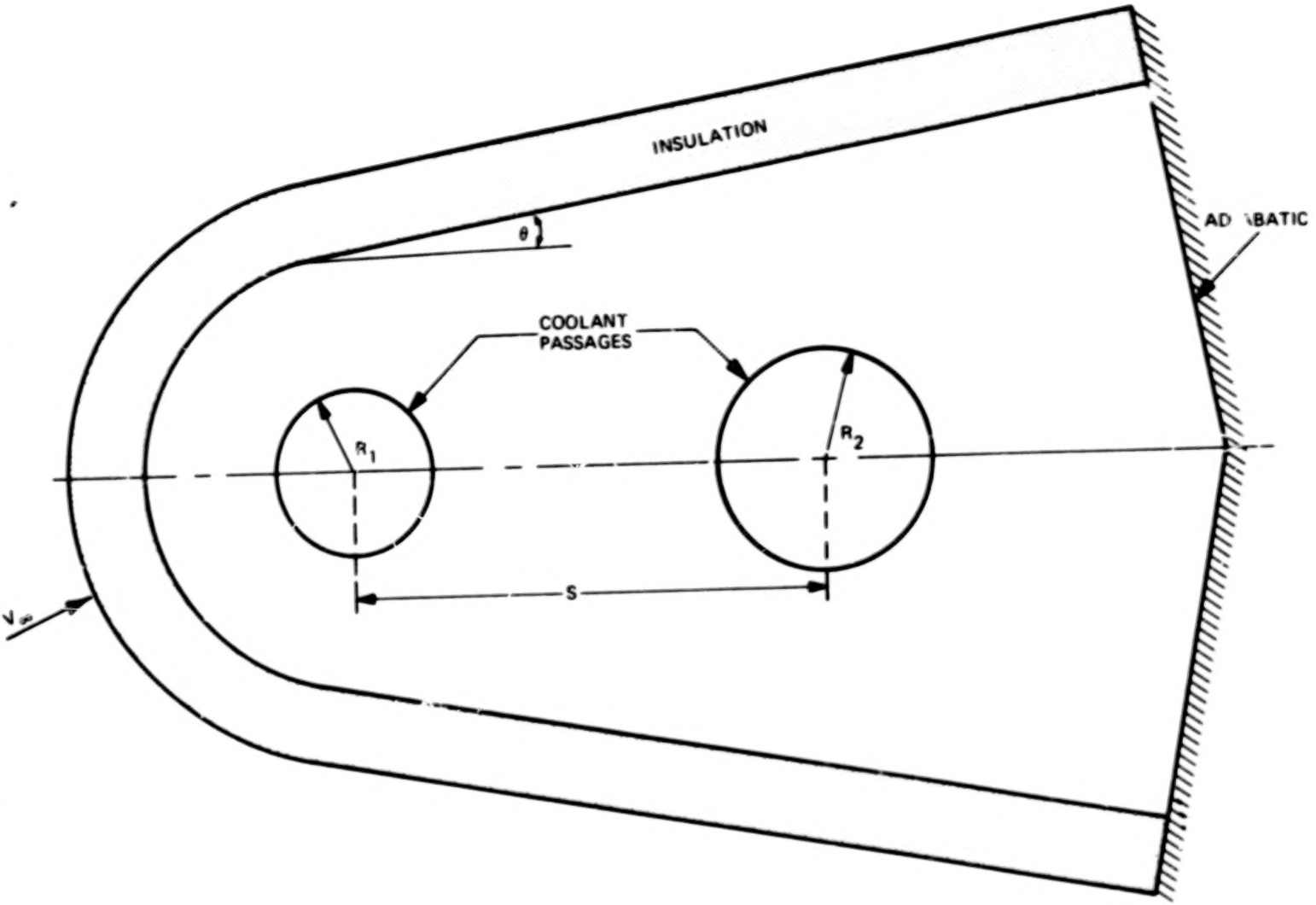


FIG. 2 LEADING EDGE GEOMETRY

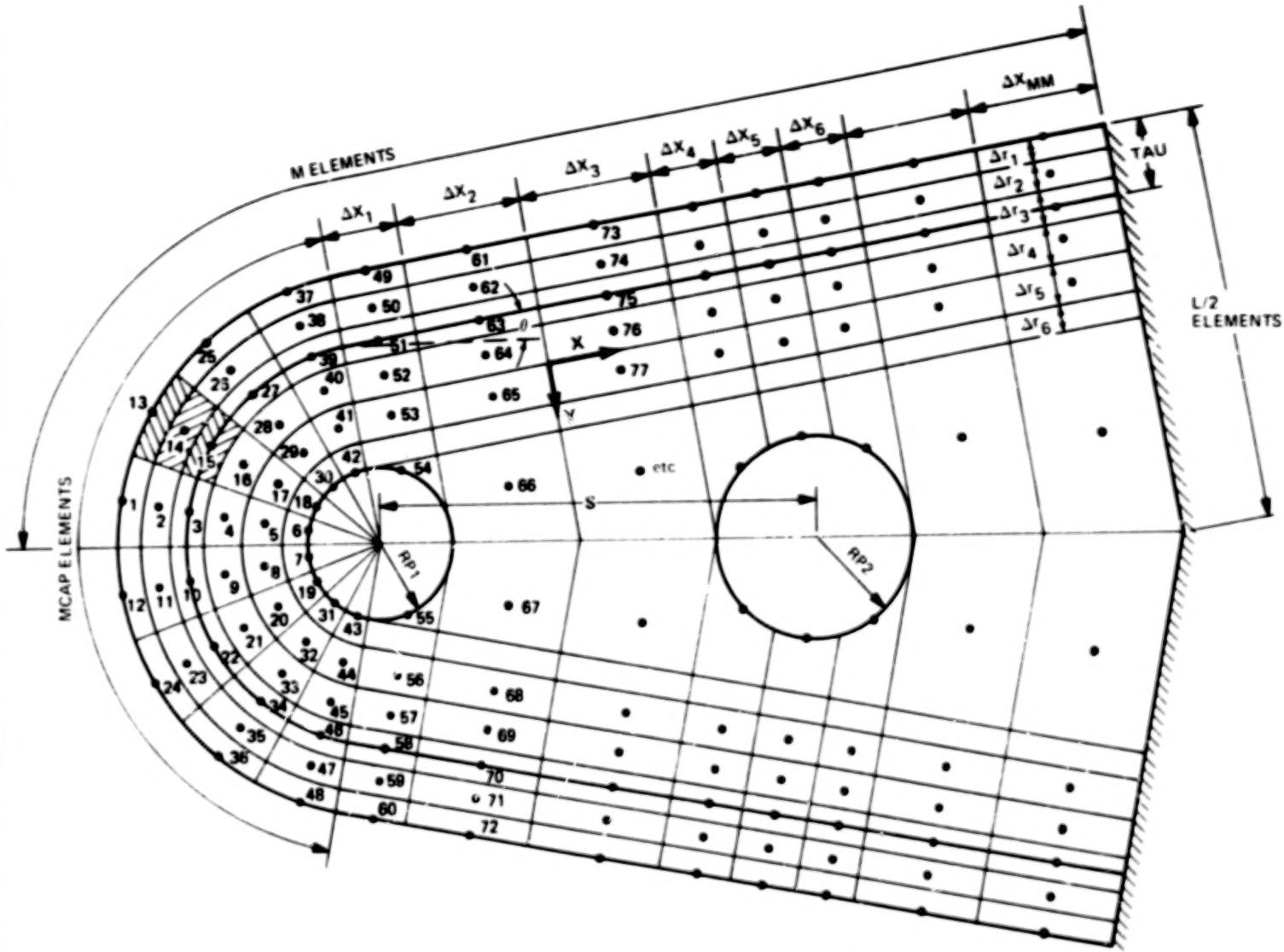


FIG. 3 GRID NETWORK FOR LEADING EDGE GEOMETRY

It is not necessary that the three ΔX's associated with the aft coolant passage be numbers 4, 5, and 6; that is just how it worked out in Figure 3. If there is no nose coolant passage, then ΔX_1 can be arbitrary in size (but nonzero).

The thickness of the insulating material is given by TAU and it may equal zero. Notice in Figure 3 that nodes are located at this interface between the two materials. LEAD4 assumes that there are equal volumes of the two materials associated with each interface node. Meaning that one half of $\Delta\tau_3$, in this case, is associated with the insulator and the other half with the main material.

The user may elect to have CAVE calculate the convective heat transfer coefficients and adiabatic wall temperatures around the leading edge, or he may supply tabular inputs for them. If the user elects the former option, then he supplies tabular values for the flight parameters of velocity, altitude and angle of attack as functions of time; moreover, he flags CAVE to use either the turbulent or laminar flow correlations, the details of which are presented in Appendix B.

For leading-edge problems that involve increased heating due to local interference heating or some other effect such as plume impingement during a portion of the flight trajectory, the user may input two tables into CAVE. The tabular values are multiplicative factors which are position and time dependent. Values from one table are used to modify the convective coefficient on the top surface and values from the other table are for the bottom surface. A nonzero value for the input variable HMODI flags CAVE that this heating multiplier option will be exercised. For the normal run when the convective coefficient is not to be modified, HMODI equals 0, and the tables are omitted.

In using this multiplier option, it is important to bear in mind that in modifying the convective coefficients, CAVE takes the average of the multiplicative factor at the beginning and end of the time interval and applies it over the entire interval. Therefore, the tables and computing time intervals must be selected with same care whenever step changes are to be simulated. A sample problem in Section 4 illustrates this.

The node numbering scheme for the leading edge geometry is interesting. A review of Figure 3 shows the following pattern:

When we get to the sample problem, we shall see that the temperatures and other nodal properties are printed out in the following array form:

1●	13●	25●
2●	14●	26●
3●	15●	etc.
4●	16●	
5●	17●	
6●	18●	
7●	19●	
8●	20●	
9●	21●	
10●	22●	
11●	23●	
12●	24●	

Two observations can be made. First, the nodes along the top surface of the leading edge (numbers 1, 13, 25, . . .) are printed out as the first row of the array and the nodes along the bottom surface (numbers 12, 24, 36, . . .) are printed out as the last row. And, secondly, if the elements of the first column are rotated as the arrows indicate, the array gets rearranged into something looking somewhat like the nodal arrangement within the leading edge. With a little experience, the user of CAVE is able to quickly scan the output and get an immediate feel for the temperature gradients within the leading edge.

The following subsections present the input data format for leading-edge problems as well as for a sample problem.

3.2 INPUT DATA FORMAT FOR LEADING EDGE GEOMETRY

Indexes Card

- JGEO, L, M, NE* (4I5)

  JGEO = 1 (selects leading edge geometry)

  L = number of elements through the material (must be an even integer)

  M = number of elements along top (or bottom) half of leading edge

  NE = number of dominant eigenvalues to be used in solution (e.g., a typical number is 5)

Title Card

- Run identification, comments, etc. (5A10)

Radiation Card

- EPS1, TBG1 (2F10.5)

  EPS1 = emissivity of surface

  TBG1 = background radiation temperature, °R

*The product L x M equals the total number of nodes. The current dimension statements in CAVE require that L x M $\leq$ 200 and that M $\leq$ 25 for the leading edge geometry.

Material Properties Cards

• MAT		(I5)
• NMAT1, RHO1, CONAV1, CPAV1		(I10,3F10.5)
• TMAT1, TMAT1(2), ..., TMAT1 (NMAT1)	omit	(8E10.0)
• CONDT1(1), CONDT1(2), ..., CONDT1 (NMAT1)	if	(8E10.0)
• CPT1(1), CPT1(2), ..., CPT1 (NMAT1)	NMAT1=0	(8E10.0)

(If MAT = 2 include the cards:)

• NMAT2, RHO2, CONAV2, CPAV2		(I10.0,3F10.5)
• TMAT2(1), TMAT2(2), ..., TMAT2 (NMAT2)	omit	(8E10.0)
• CONDT2(1), CONDT2(2), ..., CONDT2 (NMAT2)	if	(8E10.0)
• CPT2(1), CPT2(2), ..., CPT2 (NMAT2)	NMAT2=0	(8E10.0)

MAT = number of materials (1 or 2)

NMAT1 = number of entries in properties table (maximum of 10). NMAT1 = 0 for constant properties

RHO1 = density of material 1, lbm/cu-ft

CONAV1 = average thermal conductivity of material 1 (used when NMAT1 = 0), Btu/ft-sec-°R

CPAV1 = average specific heat of material 1 (used when NMAT1 = 0), Btu/lbm-°R

TMAT1 (I) = temperatures in thermal properties table for which CONDT1 (I) and CPT1 (I) are given; I = 1, 2, ..., NMAT1, °R

CONDT1 (I) = thermal conductivity of material 1 at temperature TMAT1 (I), Btu/ft-sec-°R

CPT1 (I) = specific heat of material 1 at temperature TMAT1 (I), Btu/lbm-°R

NMAT2, RHO2, CONAV2, etc., same as NMAT1, RHO1, CONAV1, etc., except applied to material 2

Detail Geometry Cards

- MCAP, THETA (I10, F10.5)
- DELX(1), DELX(2), DELX(3), ..., DELX(MM) (8F10.5)
- DELR(1), DELR(2), DELR(3), ..., DELR(L/2) (8F10.5)
- TAU (F10.5)
- RP1, RP2, S, HCOOL1, HCOOL2, TCOOL1, TCOOL2 (7F10.5)

MCAP = number of elements into which nose of leading edge is subdivided (must be an even integer)

THETA = wedge half angle of leading edge, in degrees

DELX(I) = spatial increments in x direction I=1, 2, ..., MM (where MM = M - MCAP/2), ft

DELR(I) = spatial increments in radial direction I=1, 2, ..., L/2

TAU = thickness of material 1, ft (when considering only one material, TAU = 0)

RP1 = radius of nose coolant passage, ft

RP2 = radius of aft coolant passage, ft

S = distance between coolant passage centers, ft

HCOOL1 = convective heat transfer coefficient inside nose coolant passage, Btu/ft^2-sec-°R

HCOOL2 = convective heat transfer coefficient inside aft coolant passage, Btu/ft^2-sec-°R

TCOOL1 = nose coolant temperature, °R

TCOOL2 = aft coolant temperature, °R

Initial Temperature Cards

- KODE, I, T(I), II, JJ (2I5, E10.0, 2I5)
- ...
- ...
- ...
- 11100 (indicates end of initial temperature cards) (I5)

KODE = 0 or blank

I = node number

T(I) = node initial temperature, °R

II and JJ = the node number is incremented by the spacing JJ until the limit II is reached. Each node so specified is assigned the same temperature

Wing Angles Card

- SWEEPA, DIHEDA, CODEX, HMODI, TURBL (5E10.0)

SWEEPA = wing sweep angle, in degrees

DIHEDA = wing dihedral angle, in degrees

CODEX = 0. for convective coefficient and adiabatic wall temperature computed by CAVE; = -1. for tabular input of coefficients and temperatures

HMODI = nonzero value indicates that two tables will be read at the end and used to multiply the convective coefficient

TURBL = 0. for laminar flow, = 1. for turbulent flow

Air Properties Card (omit when CODEX = -1.)

- GAM, RGAS, PR (3E10.0)

GAM = ratio of specific heats of air

RGAS = gas constant for air, ft-lbf/lbm °R

PR = Prandtl number of air

Flight Trajectory Cards (omit when CODEX = -1.)

- NTRAJ (I10)
- TIMTAB(1), TIMTAB(2), ..., TIMTAB(NTRAJ) (8E10.0)
- ALTTAB(1), ALTTAB(2), ..., ALTTAB(NTRAJ) (8E10.0)
- VELTAB(1), VELTAB(2), ..., VELTAB(NTRAJ) (8E10.0)
- ALPTAB(1), ALPTAB(2), ..., ALPTAB(NTRAJ) (8E10.0)

NTRAJ = number of points in trajectory table ($2 \leq NTRAJ \leq 50$)

TIMTAB(I) = time in trajectory table I=1, 2, ..., NTRAJ, sec

ALTTAB(I) = altitude corresponding to time TIMTAB(I), ft

VELTAB(I) = velocity corresponding to time TIMTAB(I), ft/sec

ALPTAB(I) = angle of attack corresponding to time TIMTAB(I) must be non-negative, degrees

Convective Coefficient and Adiabatic Wall Temperature Cards (omit when CODEX = 0.)

The input format for these four tables is described under subroutine NURED given in Appendix D, Sheet D-1. (Note that in the structure of these tables time is considered argument 1 and distance argument 2.)

- Table of convective coefficient as a function of time and distance along top of leading edge
- Table of adiabatic wall temperature as a function of time and distance along top of leading edge (note that the tabular entries of this table for time and distance must be identical to those of the above table)
- Table of convective coefficient as a function of time and distance along bottom of leading edge
- Table of adiabatic wall temperature as a function of time and distance along bottom of leading edge (note that the tabular entries of this table for time and distance must be identical to those of the above table)
- Blank card (terminates table read in)

Time Intervals Cards

- NTIMES (I10)
- TIMES(1), TIMES(2), ... TIMES (NTIMES) (8E10.0)

NTIMES = number of points in time intervals array ($2 \leq NTIMES \leq 50$)
TIMES(1) = initial time (usually equals 0.), sec
TIMES(I) = time at which temperatures will be calculated and printed out I = 2, 3, ..., NTIMES, sec

Convective Coefficient Modification Tables (omit when HMODI = 0.)

Two tables are required to modify the convective heat transfer coefficient (see p. 14). The first table gives the multiplicative factors for the top surface of the leading edge; the second table gives the factors for the bottom surface. Time is considered argument 1 and distance argument 2. The writeup for subroutine NURED, Appendix D (Sheet D-1), gives the specifics on the format requirements. Follow these two tables with a blank card; omit the blank card if the tables are not read in.

3.3 SAMPLE PROBLEM FOR LEADING EDGE GEOMETRY

This subsection contains an illustration of the leading edge geometry considered as a sample problem (see Fig. 4). The main features are that it is made of beryllium, has a nose radius of 0.52 cm and is cooled internally via nose and aft coolant passages. The trajectory is one of a missile with a maximum Mach number of six.

Reference to the listing of the input data (see Sheets 3.3 on p. 24) shows the time step intervals used in this problem were: 0 to 10, 10 to 20, 30 to 40.4, 40.4 to 80, and 80 to 120 seconds. Smaller time steps were used in the beginning because the trajectory is changing more rapidly then. It should be noted that for a problem such as this one where radiation is neglected and the thermal properties are considered constant, it is possible to use one single time step to cover any portion of the trajectory with constant flight parameters. Specifically, since the flight parameters velocity, altitude, and angle of attack are constant from 40.4 to 120 seconds, it would have been possible to use the single interval 40.4 to 120 in lieu of the two intervals 40.4 to 80 and 80 to 120 seconds. For laminar flow where the convective coefficient is independent of wall temperature, CAVE would calculate the same temperatures at time 120 seconds either way since the boundary conditions and properties are constant throughout the interval. The 80 second point was introduced here so as to obtain a printout of the temperatures at this time for plotting purposes.

The following pages show listings of the input data and the output generated by CAVE for this leading edge problem. The sequence of the output is:

- Statement regarding storage allocated for S array in main program
- Geometry related input data
- Node numbers adjacent to exterior boundary, nose cooling passage and aft cooling passage
- Material properties
- Trajectory table
- Node number location within output array
- Material number assigned to each node. (In this problem there is only one material being used.)

- The capacitance of each node
- The conductance in the x-direction between nodes
- The conductance in the y-direction between nodes
- Initial temperature distribution

And then the following information is printed for each time interval:

- Flight trajectory parameters, Mach number, altitude, velocity, and angle of attack at the end of the time interval
- Average heat transfer coefficients calculated using the h values at the beginning and end of this time interval
- Average heat transfer couplings, which include radiation effects, if any, calculated using the temperatures at the beginning of this time interval (See Appendix C)
- Average adiabatic wall temperatures for this time interval
- Temperatures at the end of the interval
- Steady-state temperatures for the heat transfer couplings and adiabatic wall temperatures of this interval
- Integrated heat input to each node. This gives the net heat transfer at each boundary node up to the end of this time interval

(Annotation has been added to the input and output to aid the reader.)

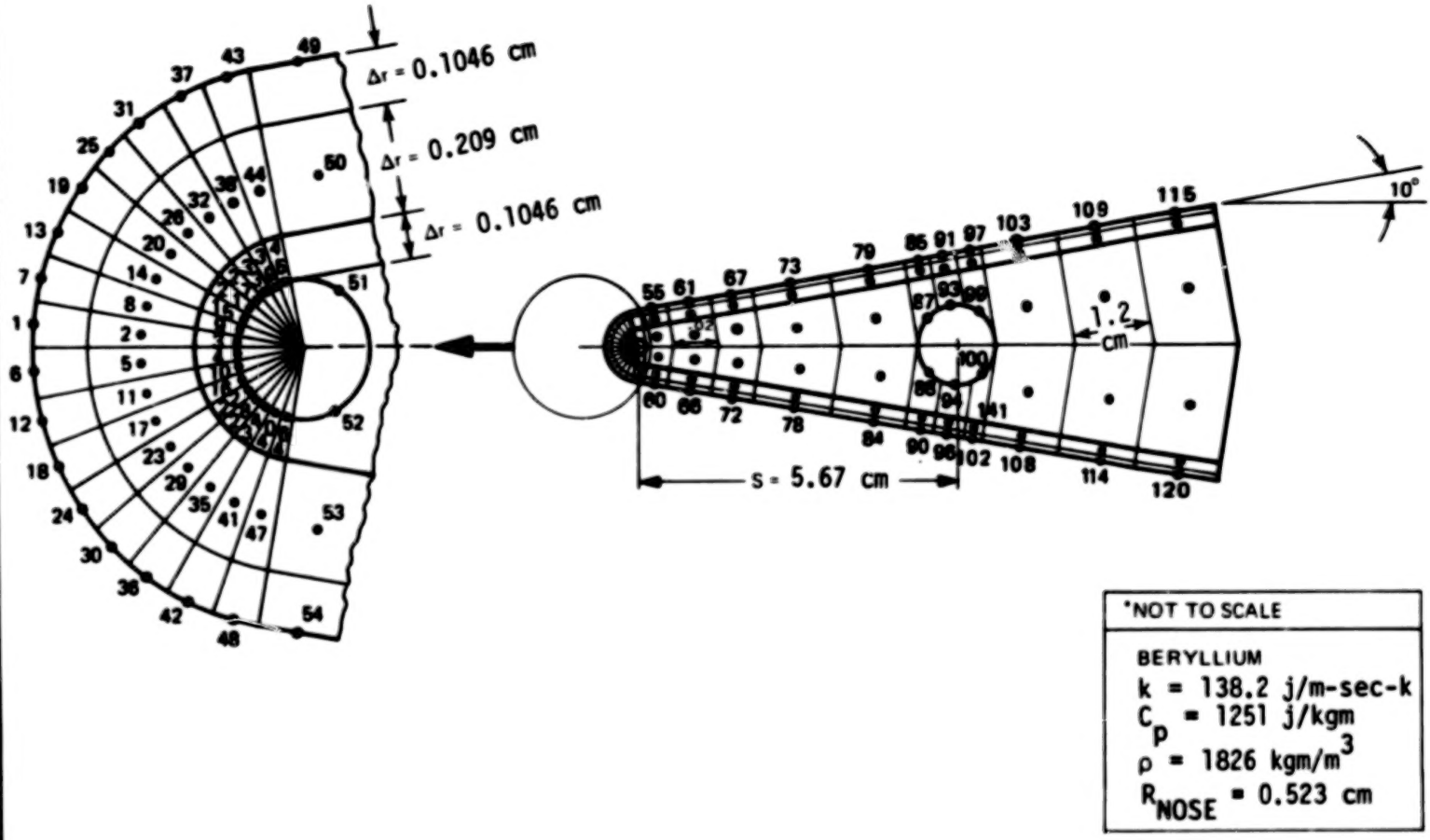
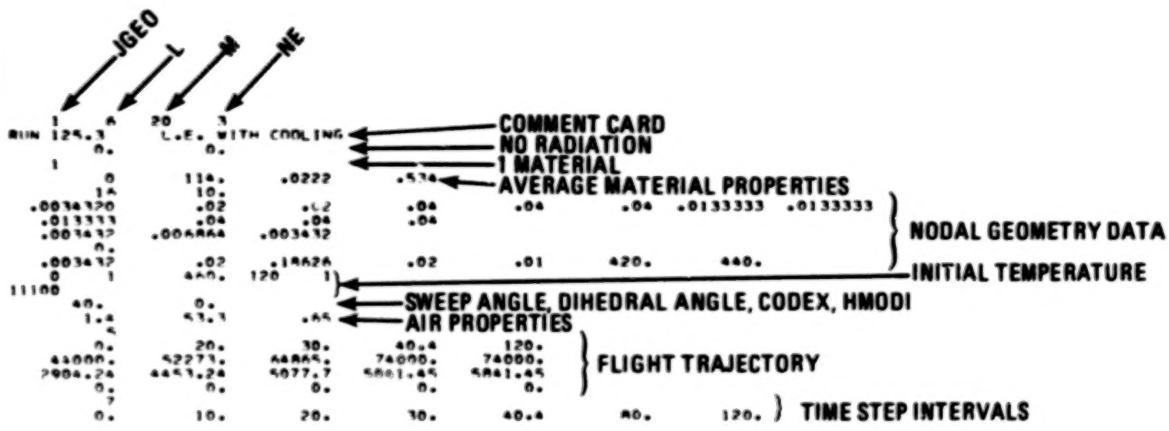
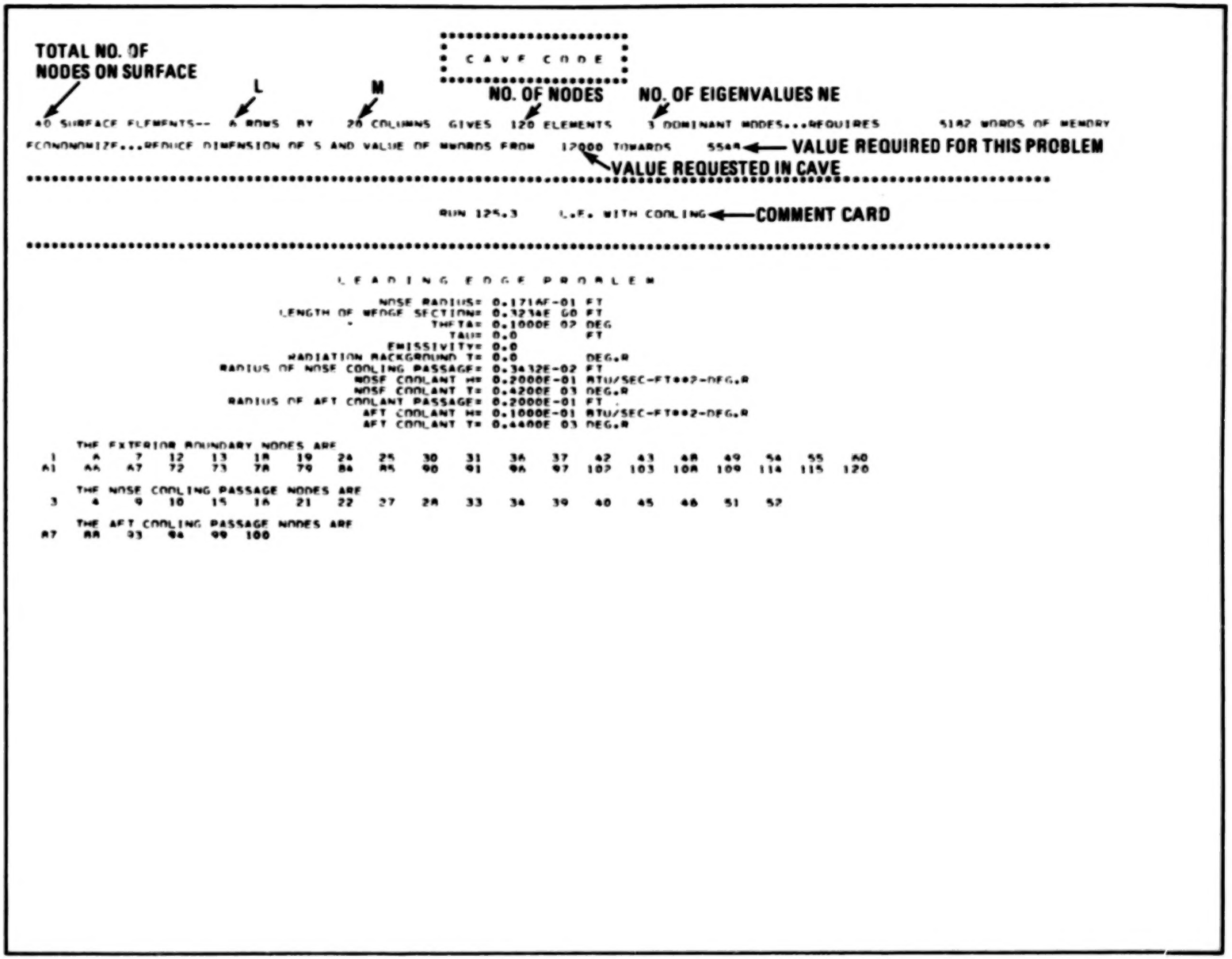


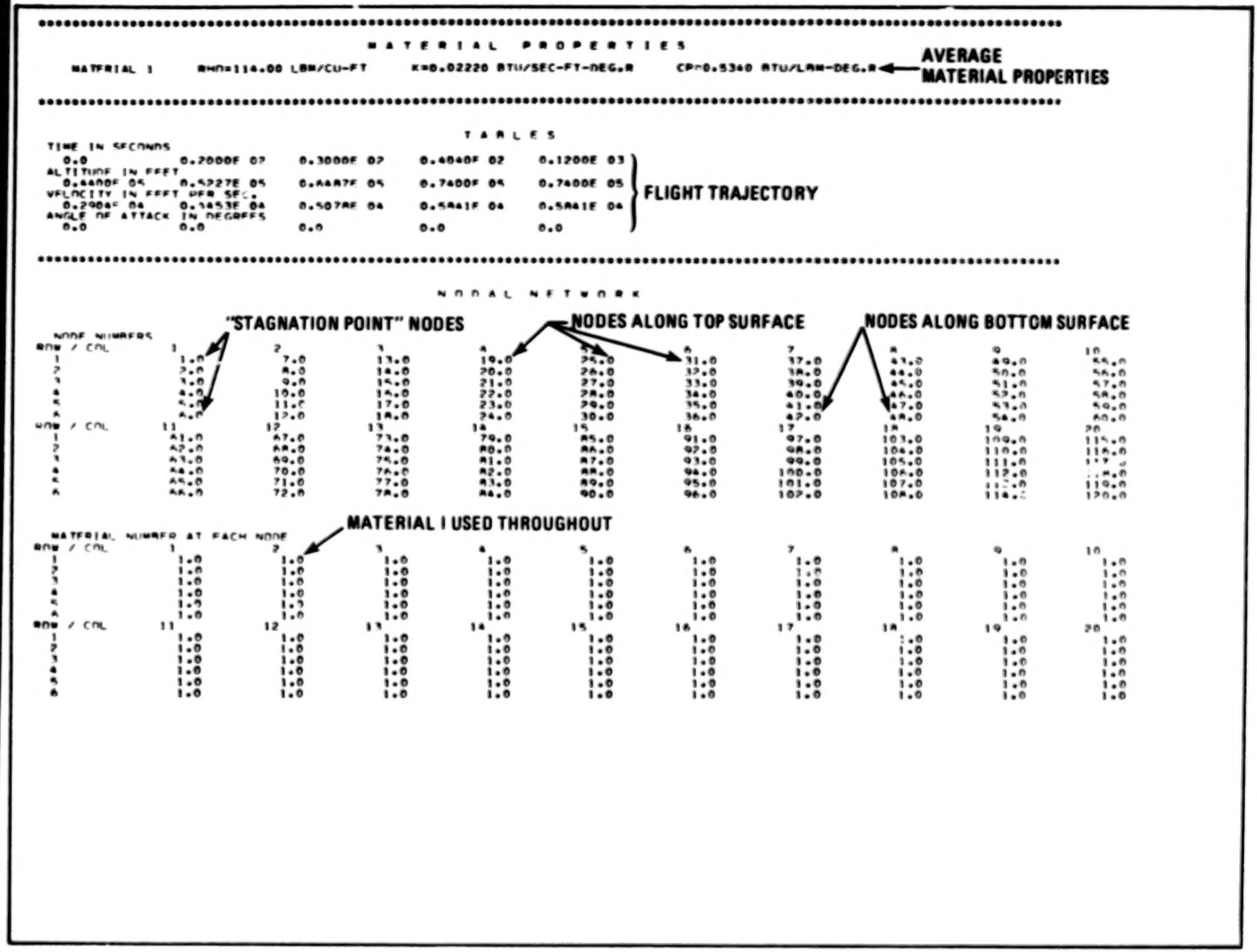
FIG. 4 GRID NETWORK* FOR MISSILE LEADING EDGE WITH COOLING



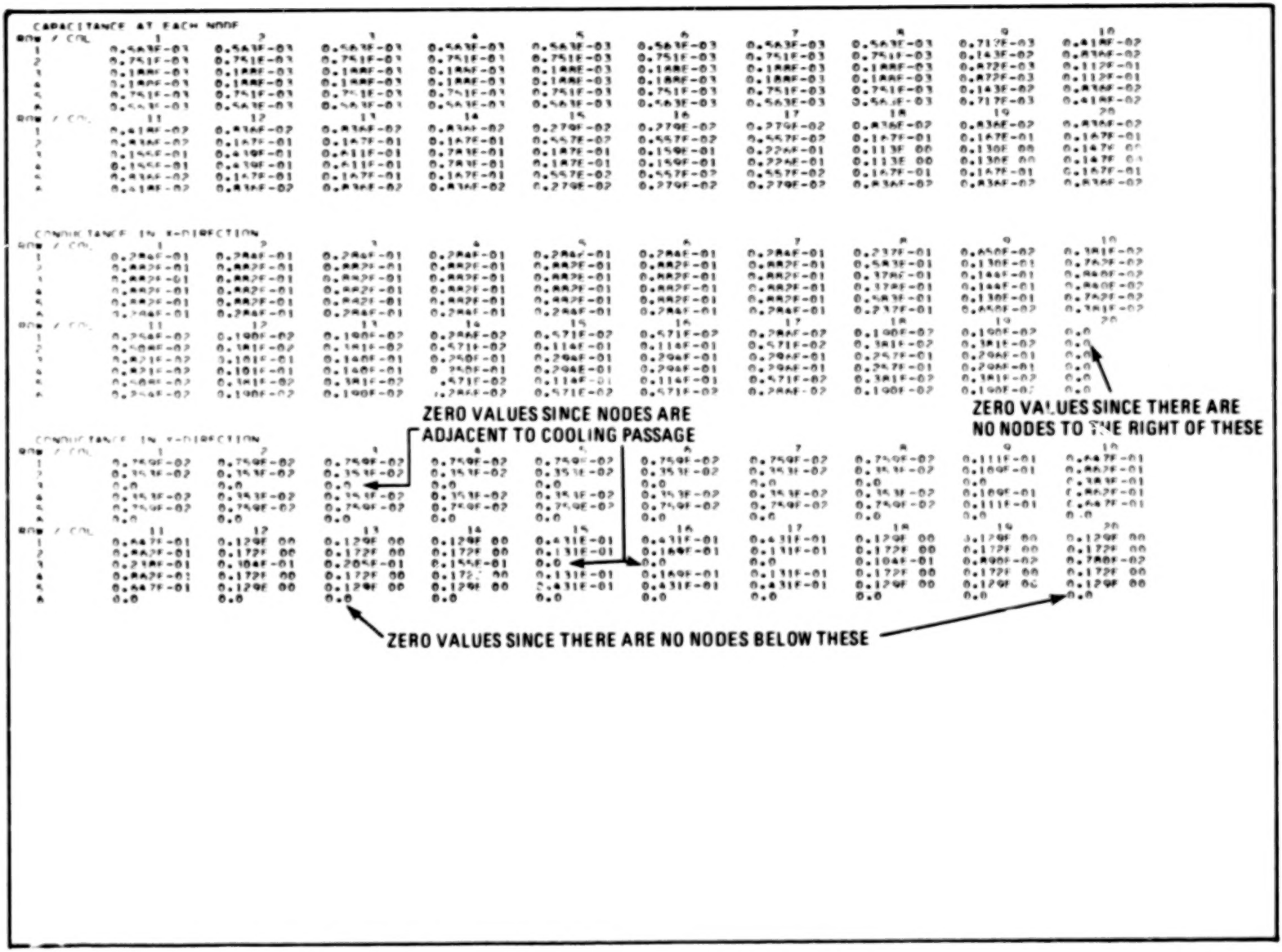
SHEET 3.3 INPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4) (SHEET 1 OF 8)


TOTAL NO. OF NODES ON SURFACE
L
M
NO. OF NODES
NO. OF EIGENVALUES NE
CAVE CODE
40 SURFACE ELEMENTS-- 4 ROWS BY 20 COLUMNS GIVES 120 ELEMENTS 3 DOMINANT MODES...REQUIRES 5182 WORDS OF MEMORY
ECONONOMIZE...REDUCE DIMENSION OF S AND VALUE OF NWORDS FROM 12000 TOWARDS 5584
VALUE REQUIRED FOR THIS PROBLEM
VALUE REQUESTED IN CAVE
RUN 125.3 L.E. WITH COOLING
COMMENT CARD
LEADING EDGE PROBLEM
NOSE RADIUS= 0.171AE-01 FT
LENGTH OF WEDGE SECTION= 0.1234E 00 FT
THETA= 0.1000E 02 DEG
TAU= 0.0 FT
EMISSIVITY= 0.0
RADIATION BACKGROUND T= 0.0 DEG.R
RADIUS OF NOSE COOLING PASSAGE= 0.3432E-02 FT
NOSE COOLANT H= 0.2000E-01 BTU/SEC-FT**2-DEG.R
NOSE COOLANT T= 0.4200E 03 DEG.R
RADIUS OF AFT COOLANT PASSAGE= 0.2000E-01 FT
AFT COOLANT H= 0.1000E-01 BTU/SEC-FT**2-DEG.R
AFT COOLANT T= 0.4400E 03 DEG.R
THE EXTERIOR BOUNDARY NODES ARE
1 A 7 12 13 18 19 24 25 30 31 36 37 42 43 48 49 54 55 60
A1 AA A7 72 73 78 79 84 85 90 91 96 97 102 103 108 109 114 115 120
THE NOSE COOLING PASSAGE NODES ARE
3 4 9 10 15 16 21 22 27 28 33 34 39 40 45 46 51 52
THE AFT COOLING PASSAGE NODES ARE
A7 AB 93 94 99 100

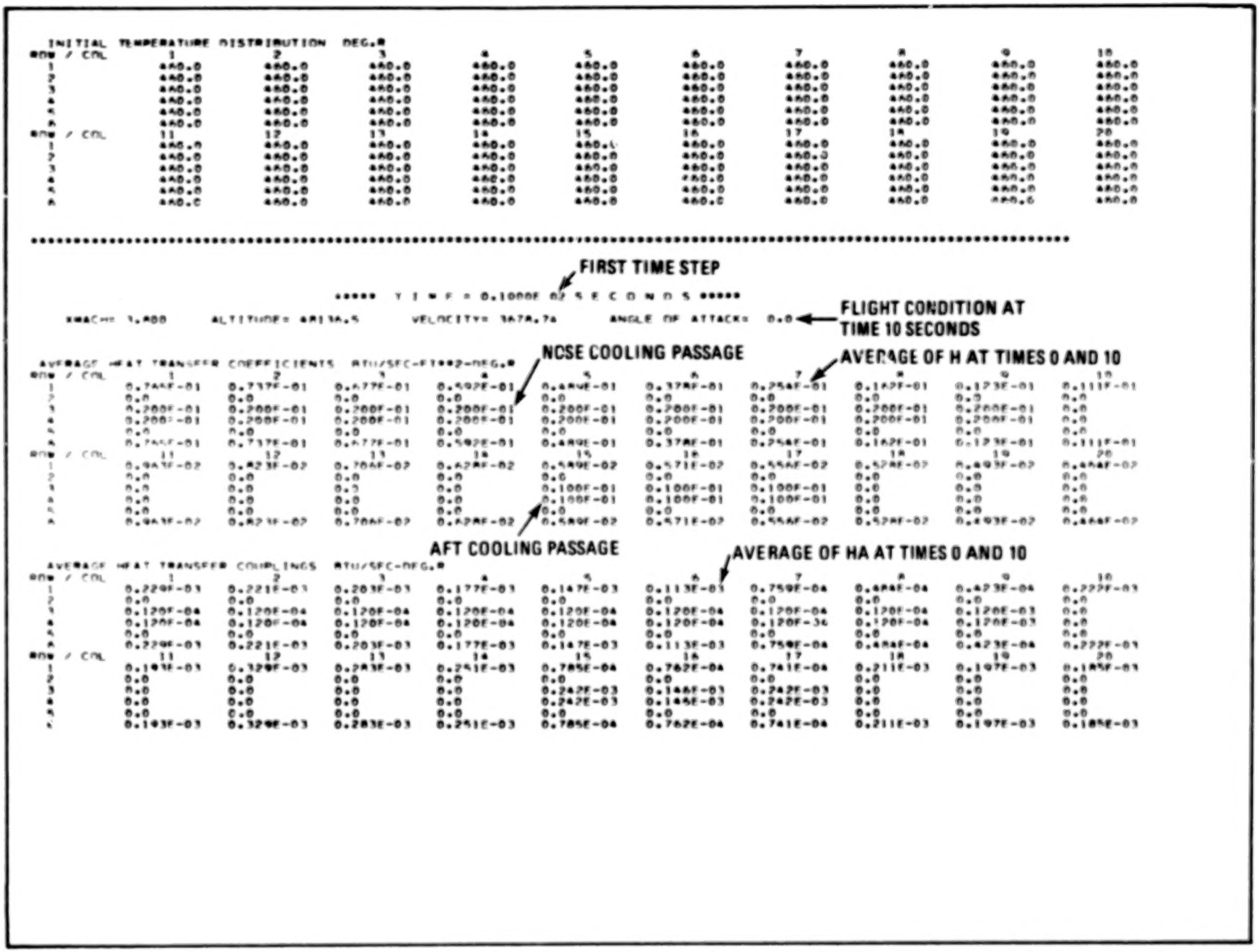
SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4)(SHEET 2 OF 8)


MATERIAL PROPERTIES
MATERIAL 1 RHO=114.00 LBM/CU-FT K=0.02220 BTU/SEC-FT-DEG.R CP=0.5340 BTU/LBM-DEG.R
AVERAGE MATERIAL PROPERTIES
TABLES
TIME IN SECONDS
0.0 0.2000E 07 0.3000E 07 0.4040E 02 0.1200E 03
ALTITUDE IN FEET
0.4400E 05 0.5227E 05 0.6487E 05 0.7400E 05 0.7400E 05
VELOCITY IN FEET PER SEC.
0.2904E 04 0.1453E 04 0.5079E 04 0.5841E 04 0.5841E 04
ANGLE OF ATTACK IN DEGREES
0.0 0.0 0.0 0.0 0.0
FLIGHT TRAJECTORY
NODAL NETWORK
"STAGNATION POINT" NODES
NODES ALONG TOP SURFACE
NODES ALONG BOTTOM SURFACE
NODE NUMBERS
MATERIAL NUMBER AT EACH NODE
MATERIAL I USED THROUGHOUT

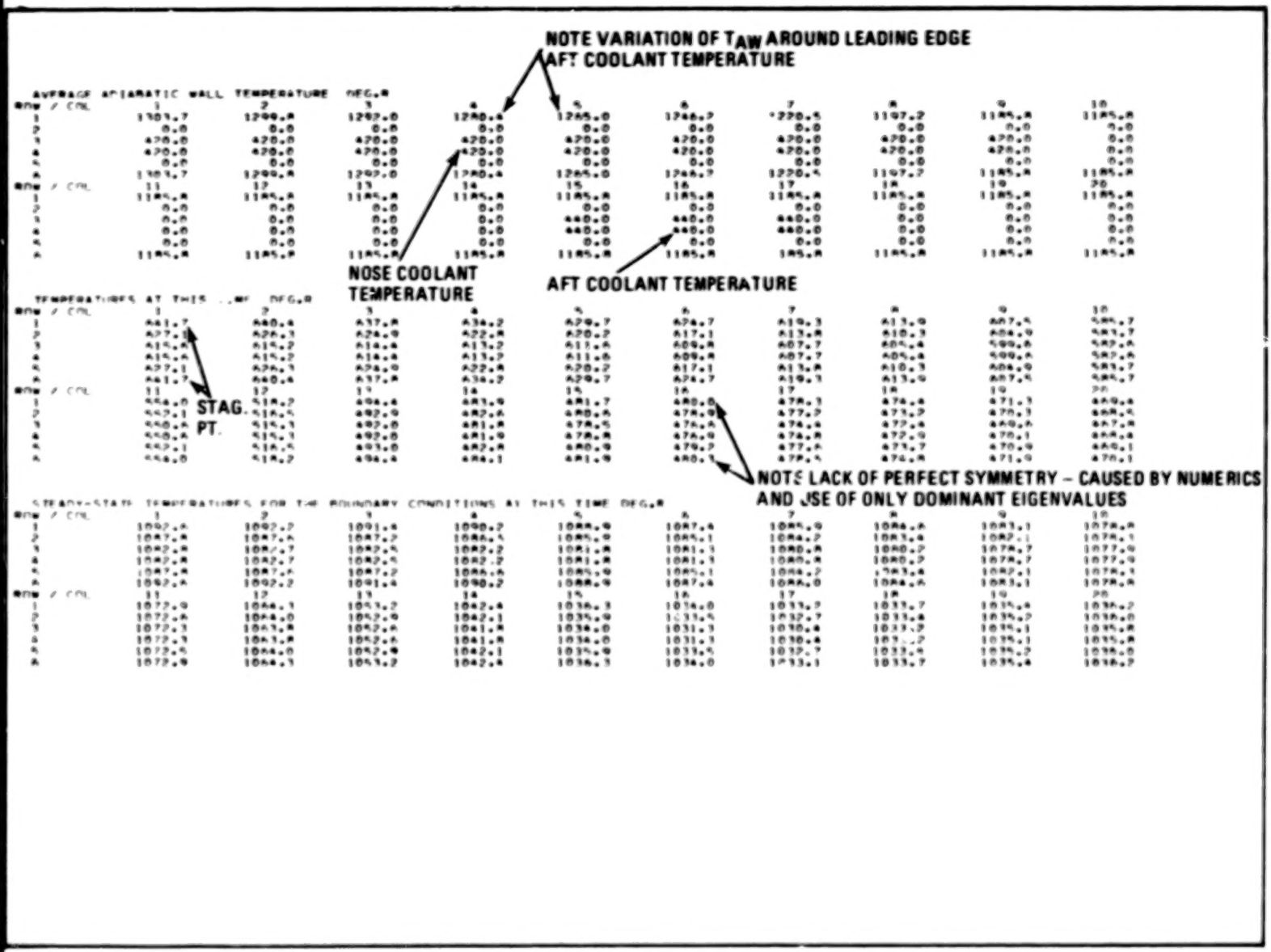
SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4)(SHEET 3 OF 8)


ZERO VALUES SINCE NODES ARE ADJACENT TO COOLING PASSAGE
ZERO VALUES SINCE THERE ARE NO NODES TO THE RIGHT OF THESE
ZERO VALUES SINCE THERE ARE NO NODES BELOW THESE

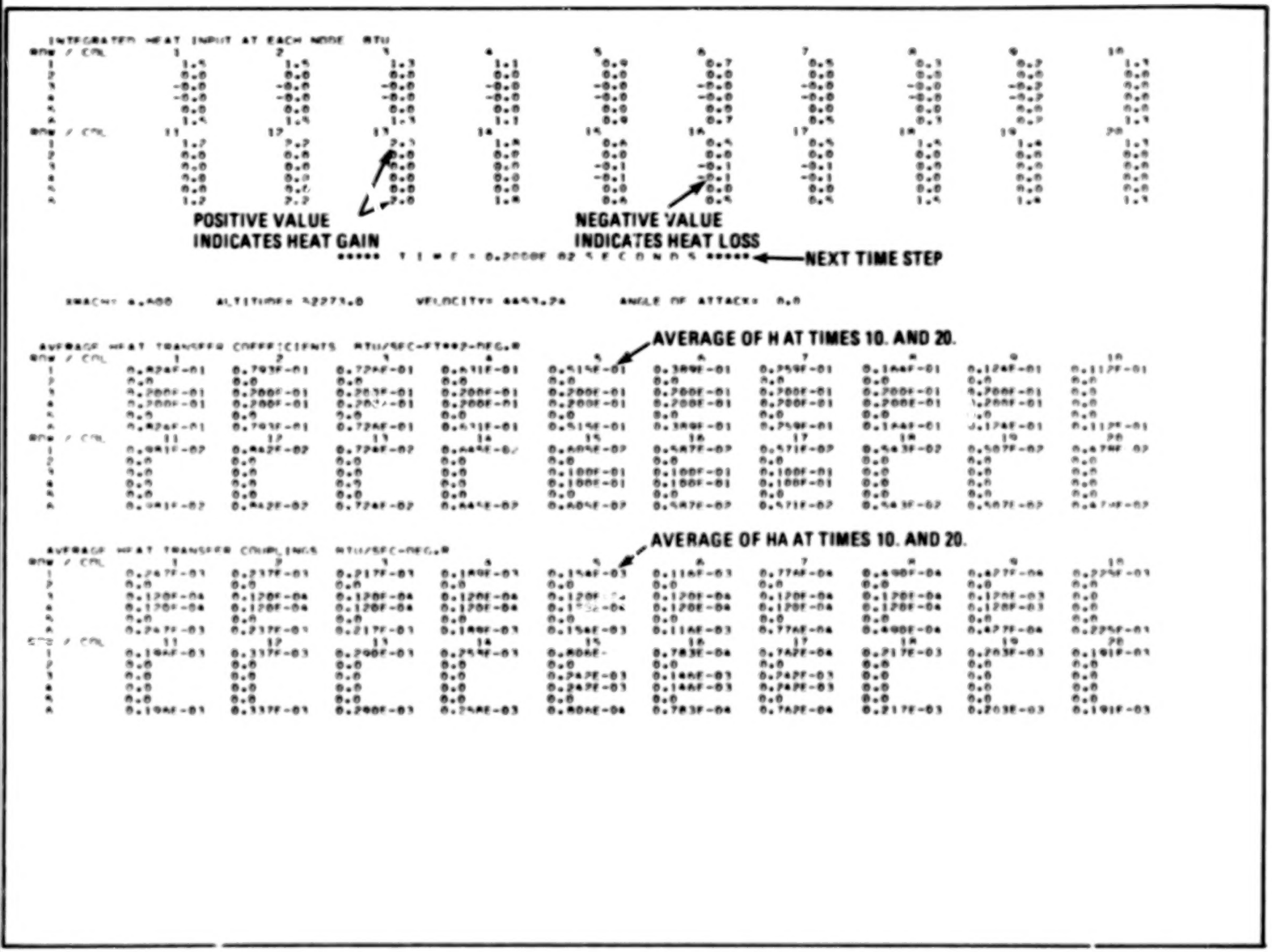
SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4) (SHEET 4 OF 8)


INITIAL TEMPERATURE DISTRIBUTION DEG.R
FIRST TIME STEP
FLIGHT CONDITION AT TIME 10 SECONDS
NOSE COOLING PASSAGE
AVERAGE OF H AT TIMES 0 AND 10
AFT COOLING PASSAGE
AVERAGE OF HA AT TIMES 0 AND 10

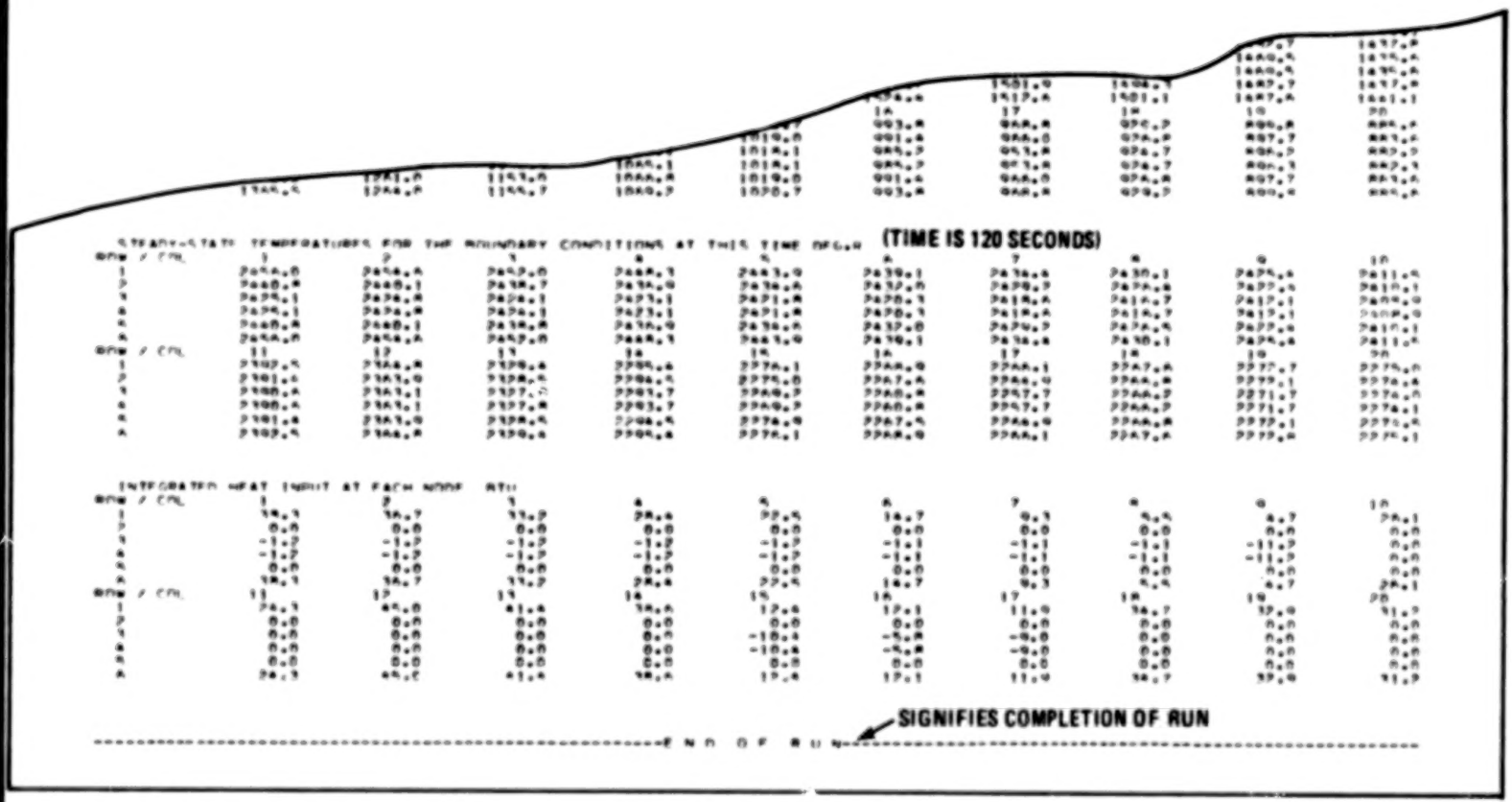
SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4)(SHEET 5 OF 8)


NOTE VARIATION OF T_{AW} AROUND LEADING EDGE
AFT COOLANT TEMPERATURE
NOSE COOLANT TEMPERATURE
AFT COOLANT TEMPERATURE
STAG. PT.
NOTE LACK OF PERFECT SYMMETRY – CAUSED BY NUMERICS AND USE OF ONLY DOMINANT EIGENVALUES

SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4) (SHEET 6 OF 8)


INTEGRATED HEAT INPUT AT EACH NODE BTU
POSITIVE VALUE INDICATES HEAT GAIN
NEGATIVE VALUE INDICATES HEAT LOSS
***** T I M E = 0.20000E 02 S E C O N D S *****
NEXT TIME STEP
MACH= 6.400 ALTITUDE= 52271.8 VELOCITY= 4693.24 ANGLE OF ATTACK= 0.0
AVERAGE HEAT TRANSFER COEFFICIENTS BTU/SEC-FT**2-DEG.R
AVERAGE OF H AT TIMES 10. AND 20.
AVERAGE HEAT TRANSFER COUPLINGS BTU/SEC-DEG.R
AVERAGE OF HA AT TIMES 10. AND 20.

SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4) (SHEET 7 OF 8)



SHEET 3.3 OUTPUT DATA FOR LEADING EDGE PROBLEM (REFER TO FIG. 4)(SHEET 8 OF 8)

Section 4

COOLED PANEL, SLAB AND X-24C STRUCTURE

4.1 DISCUSSION

This section discusses the input data format and definition of variables for the three geometries of cooled panel, slab and a structural arrangement referred to as basic X-24C structure. These geometries are shown in Figures 5 through 8 which give the overall dimensions that are input to CAVE and the type of boundary conditions associated with the geometries. For the cooled panel, there are two types of corners that can be analyzed: square and round (see Figs. 5 and 6).

The top surface of each of the three geometries experiences aerodynamic heating. The user may elect to have CAVE calculate the convective heat transfer coefficients and adiabatic wall temperatures, or he may supply tabular inputs for them. If the user elects the former option, then he supplies tabular values for the flight parameters velocity, altitude and angle of attack as functions of time.

In calculating the aerodynamic heating, CAVE assumes the boundary layer flow is processed through an oblique shock whenever the top surface is moving at speeds in excess of Mach 1 and at an angle of attack relative to the freestream conditions. Subroutine TRANS establishes whether the flow field is laminar or turbulent based on a transition criterion (see Fig. B-1, Appendix B). The details of the aerodynamic heating equations that are used are given in Appendix B.

Depending on the orientation of the geometry with respect to the boundary layer flow, the convective coefficient may vary with X (flow is from left to right in the plane of the paper) or be independent of X (flow is into plane of paper). Both conditions can be handled by CAVE. Setting the input variable CODEX equal to 1. selects the former situation, while CODEX equal to 0. selects the latter. When the convective coefficient is to be considered varying with X, the user must input a nonzero value for REFX which represents an effective boundary layer length to the left edge of the geometry (Refer to Appendix B).

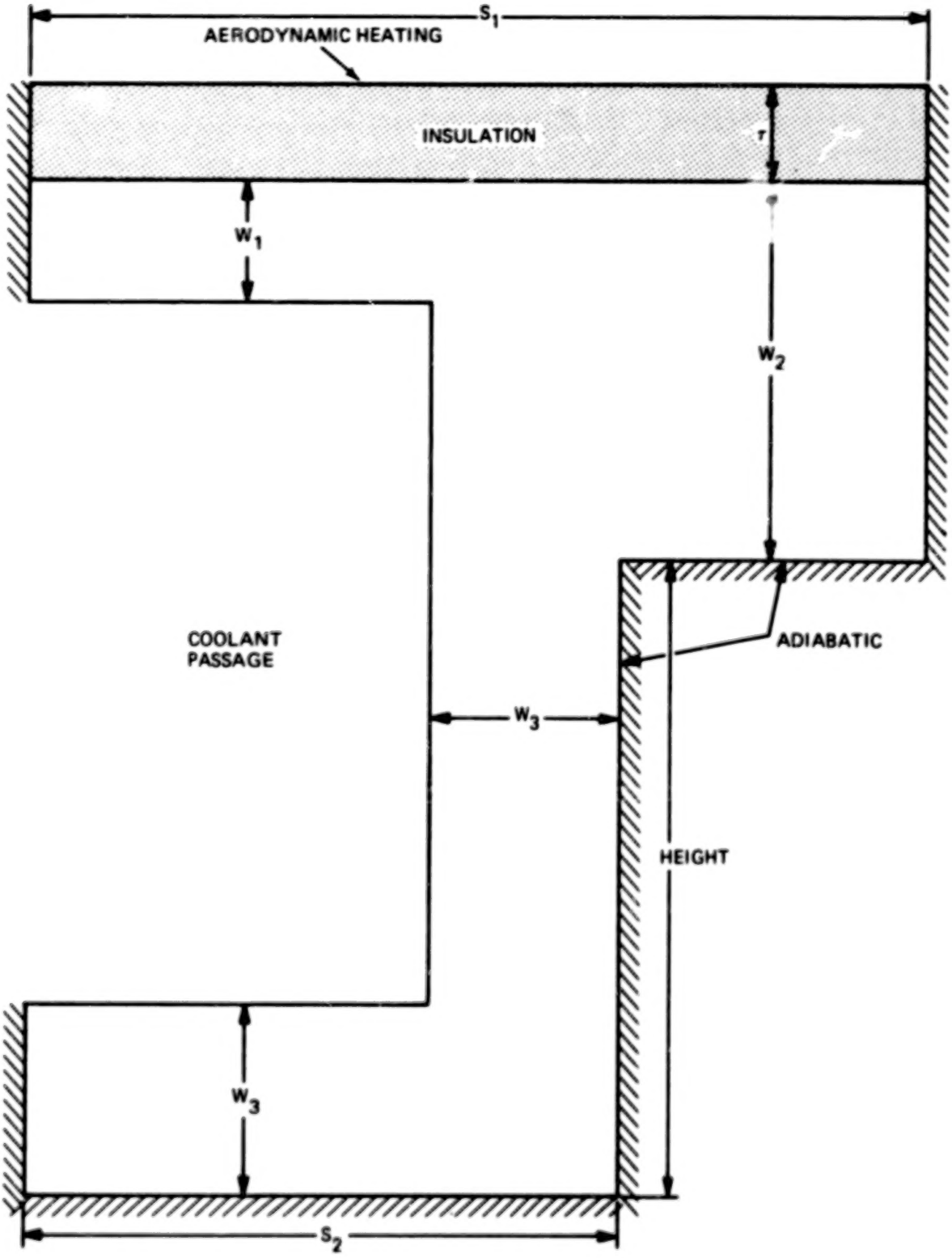


FIG. 5 COOLED PANEL GEOMETRY (SQUARE CORNER)

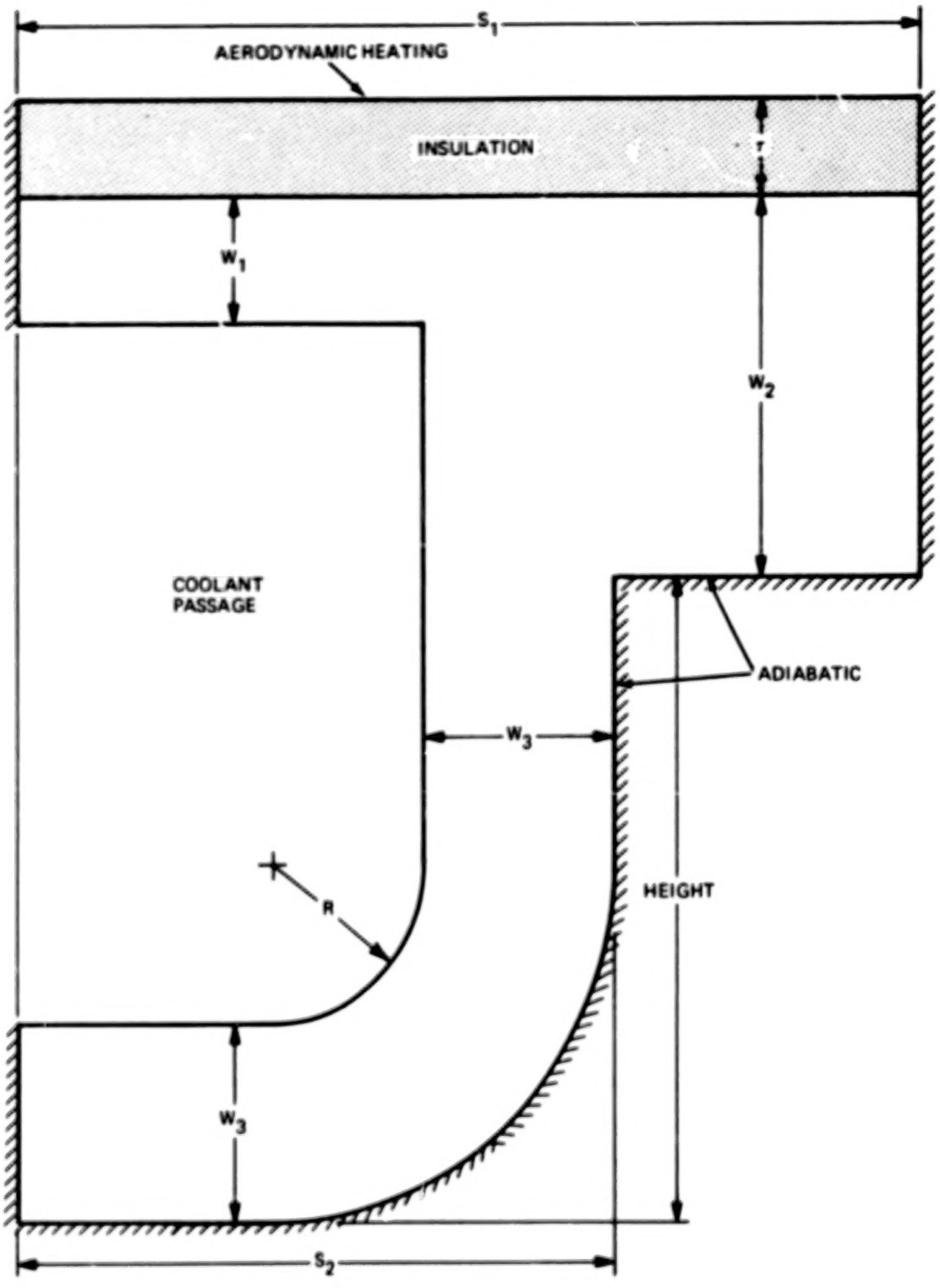


FIG. 6 COOLED PANEL GEOMETRY (ROUND CORNER)

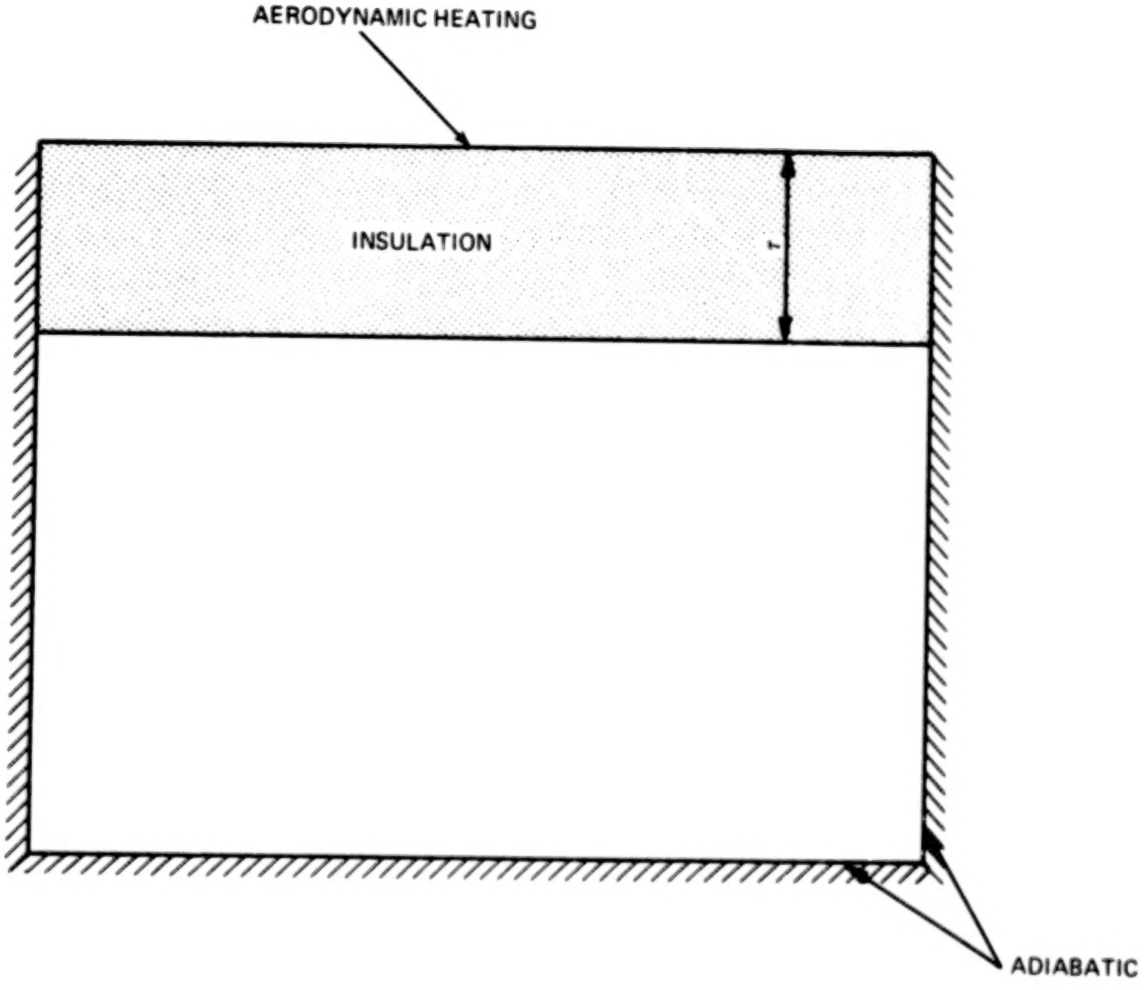


FIG. 7 SLAB GEOMETRY

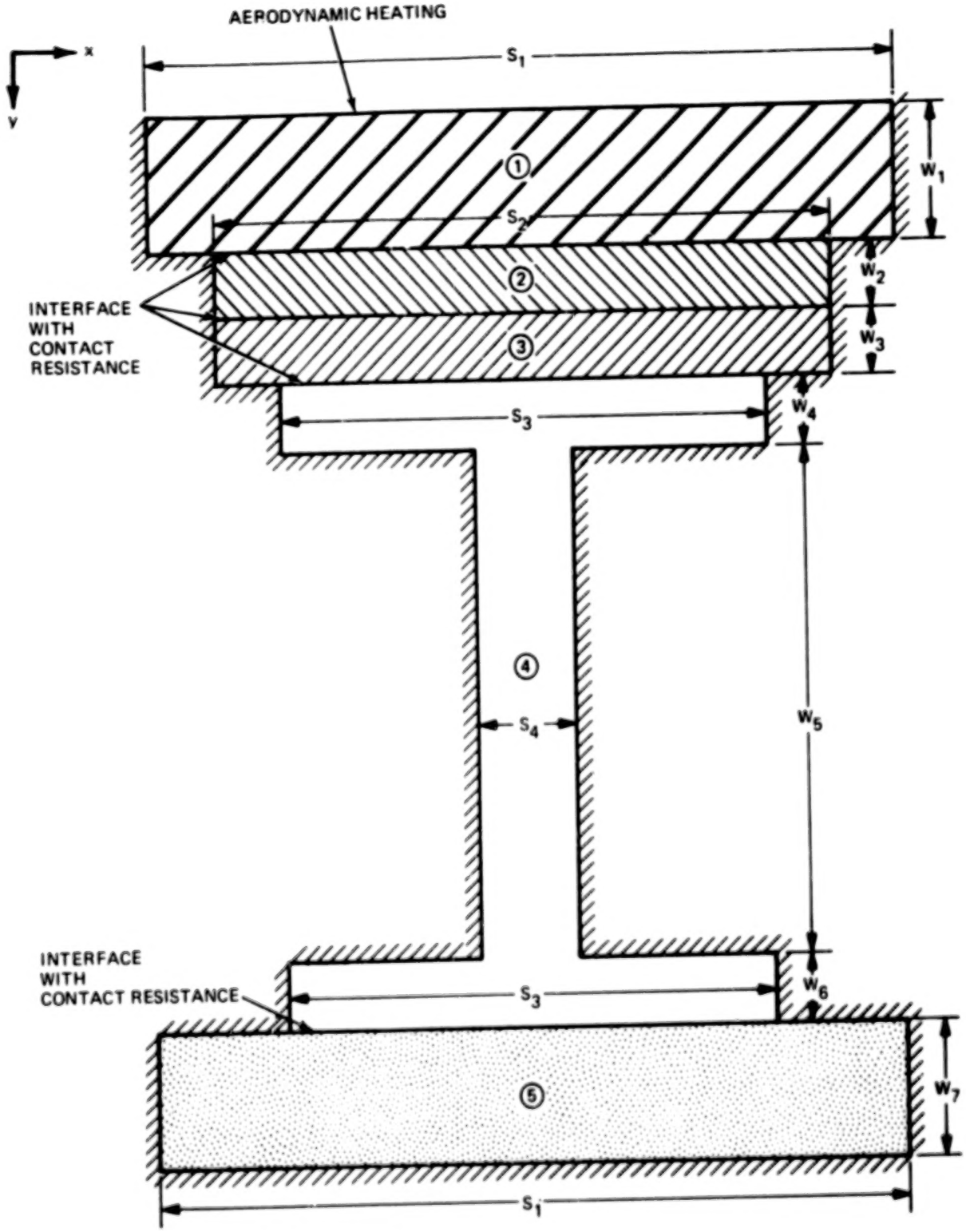


FIG. 8 X-24C STRUCTURE

As cited previously, an option exists to input directly the convective coefficient and the adiabatic wall temperatures as functions of time and X in lieu of the flight trajectory parameters. This optional feature is valuable for predicting temperatures within structural members subjected to other than the usual aerodynamic heating. The forcing functions for such special heating effects as caused by body shocks intersecting the wing, separated flow regions, wing-body interaction and engine exhaust plume impinging upon the structure can be calculated external to the code and then supplied as input data to the code. CODEX is set equal to -1. to exercise this option and the flight trajectory is not input.

An additional option exists to modify the convective coefficients by multiplicative factors which are position and time dependent. (This would be useful in accounting for shock impingement heating.) The factors are entered as a table. A nonzero value for the input variable HMODI flags CAVE that this heating multiplier option will be exercised. For the normal run when the convective coefficient is not to be modified, HMODI equals 0. and the tables are omitted.

This modification option can be exercised whether CAVE calculates the original or unmodified convective coefficients, or if they are input to CAVE.

Subroutine SLAB2 discretizes both the cooled panel and slab geometries into nodes and calculates the associated capacitances and conductances. Subroutine X-24C discretizes the X-24C geometry.

Figure 9 shows the grid network generated within SLAB2 for the cooled panel geometry of Figure 7. The network is generated based on the input dimensions S1, S2, W1, W2, W3, TAU, HEIGHT, and on the ΔX's and ΔY's (which can all be different). TAU represents the thickness of the insulating material which may equal zero. Notice that in Figure 9 nodes are located at the interface between the two materials. SLAB2 assumes that there are equal volumes of the two materials associated with each interface node. This means that one half of ΔY_3, in this case, is associated with the insulator and the other half with the main material.

Figure 10 shows the grid network generated for the cooled panel with round corner (see Fig. 6) based on the above input dimensions plus R, the inside radius of the corner.

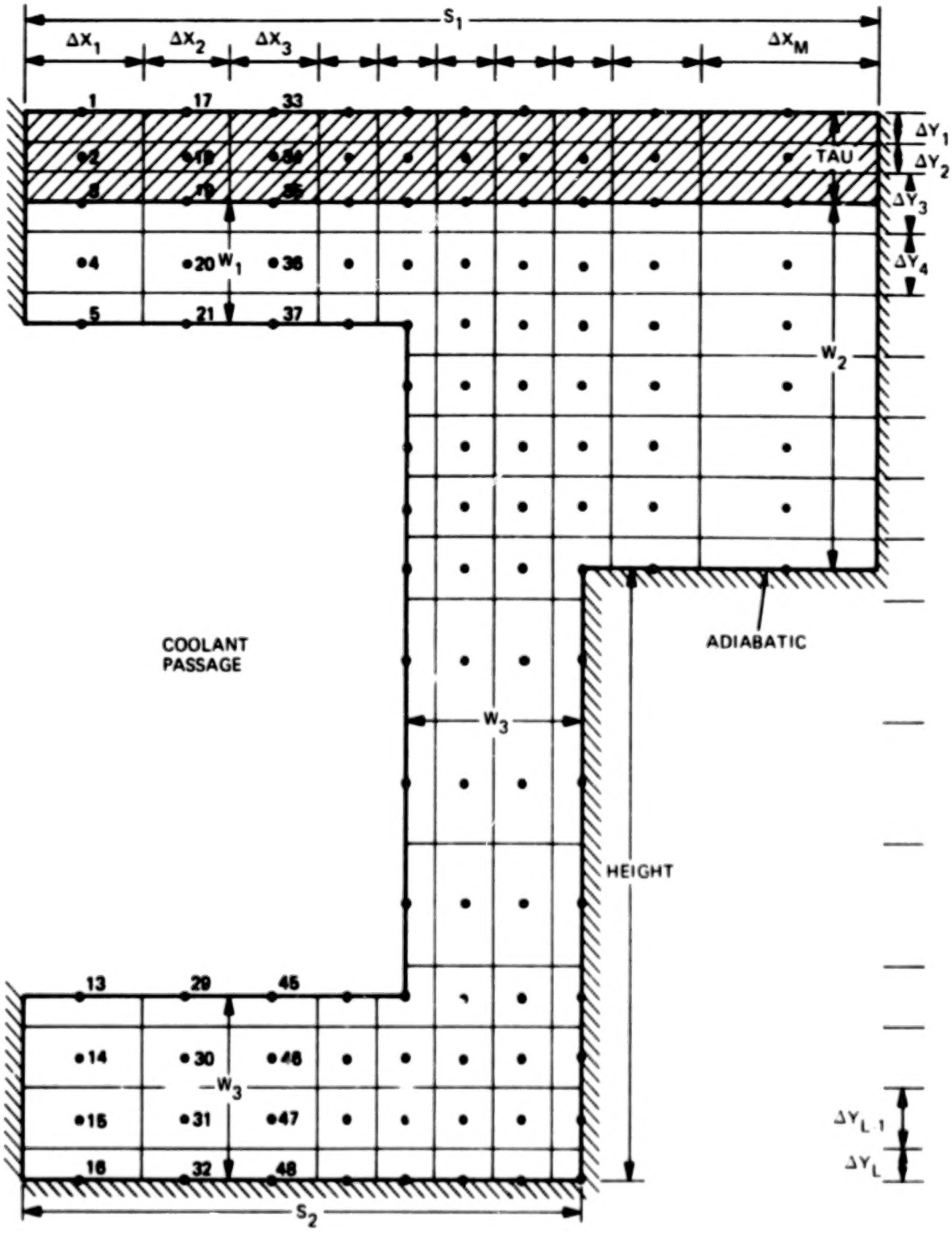


FIG. 9 GRID LAYOUT FOR COOLED PANEL (SQUARE CORNER)

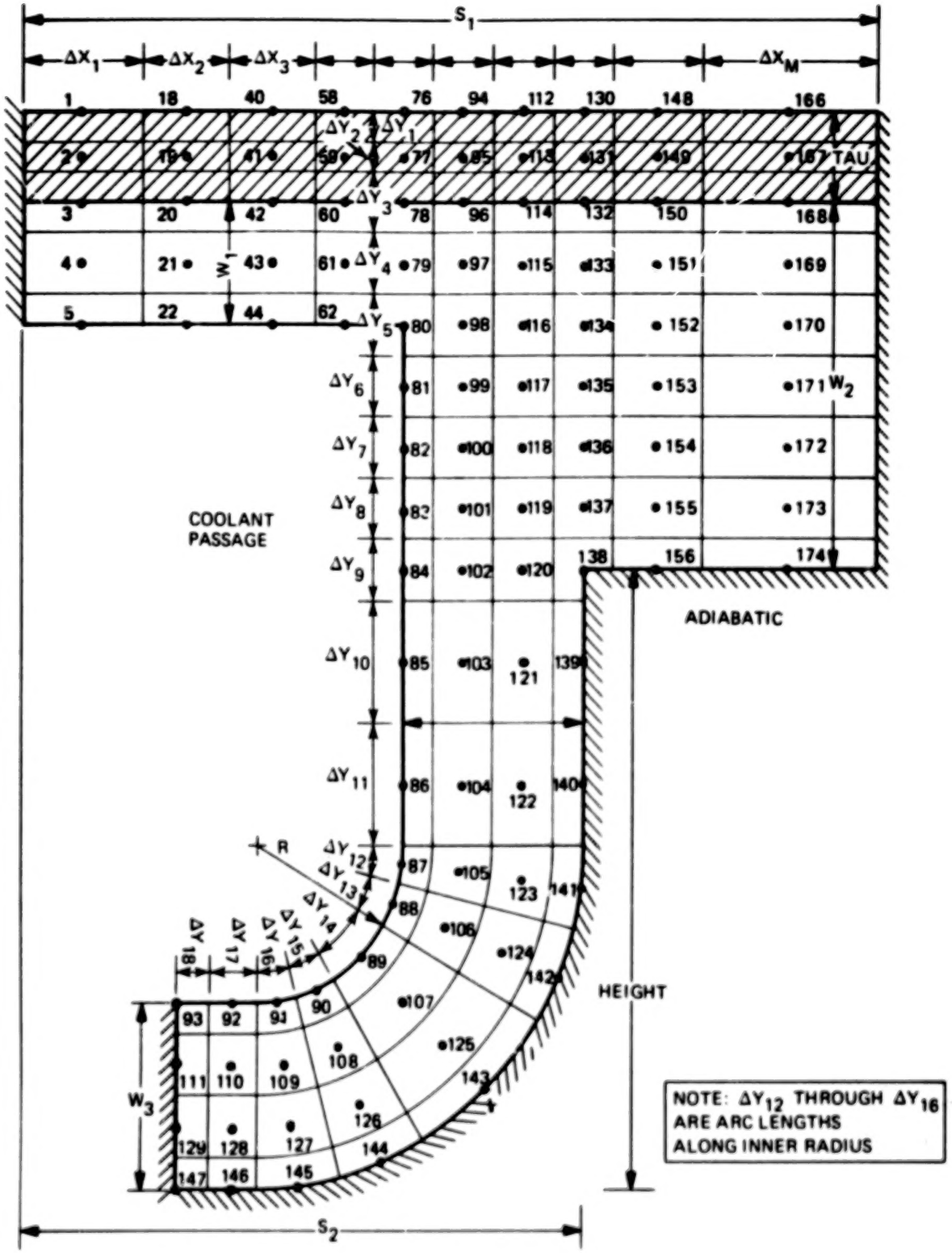


FIG. 10 GRID LAYOUT FOR COOLED PANEL (ROUND CORNER)

The convective heat transfer coefficient and the temperature of the coolant are input values. They are constant for the trajectory and may be input as zero, in which case the surface of the coolant passage is taken as being adiabatic.

Figure 11 shows the grid network generated within SLAB2 for the slab geometry of Figure 7. The network is generated based on the input values for the ΔX's and ΔY's (which may all be different). Aerodynamic heating takes place on the top surface and is handled just as in the cooled panel geometry. The other three sides of the slab are taken to be adiabatic. Slab geometry is handled very much the same as cooled panel geometry. The input card for S1, S2, W1, etc., associated with the dimensions of the cooled panel is left blank for the slab; in other cases, the required input information is the same.

A review of Figure 8 shows that the X-24C geometry introduces a feature not embodied in any of the other geometries – contact resistance between materials. The X-24C geometry can be viewed as having 5 components with a total of 4 interfaces between them. The unit surface contact resistances at the 4 interfaces are inputs to CAVE. Figure 12 gives the grid network generated within subroutine X-24C for the geometry given in Figure 8. It can be seen that nodes are located at the interfaces between components. The precise location of a node at an interface is in the upper component as the full contact resistance is applied to the conduction coupling between the interface node and the node below it. As with all the other geometries, the user must input the ΔY's such that the interfaces are mid-way within a ΔY spacing.

Another new feature to the X-24C geometry is that up to three materials can be involved in the structure instead of the usual two. There is no limitation on the arrangement of the materials among the five components. The thermal conductivity and specific heats of the materials can be constant or temperature dependent.

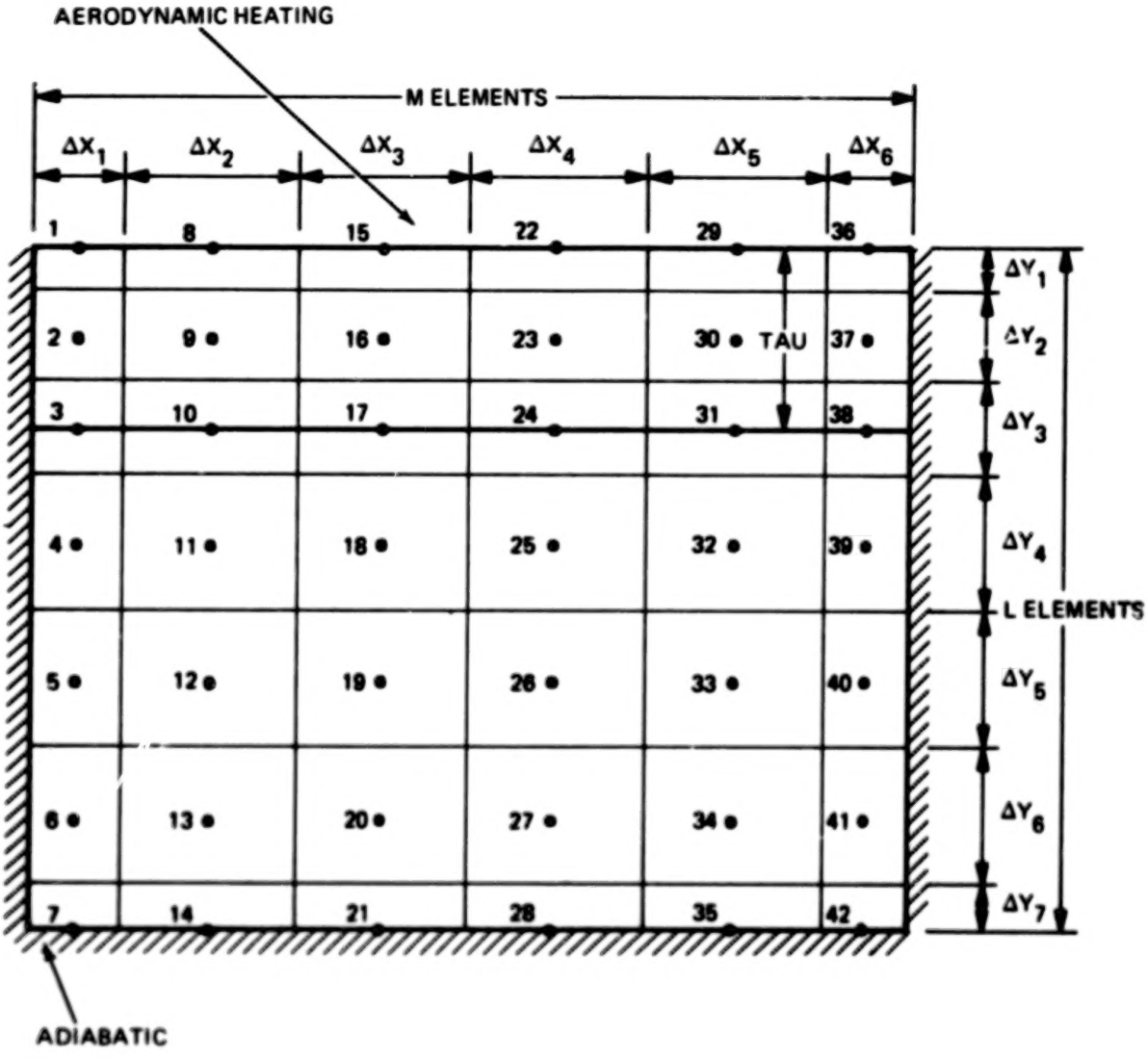
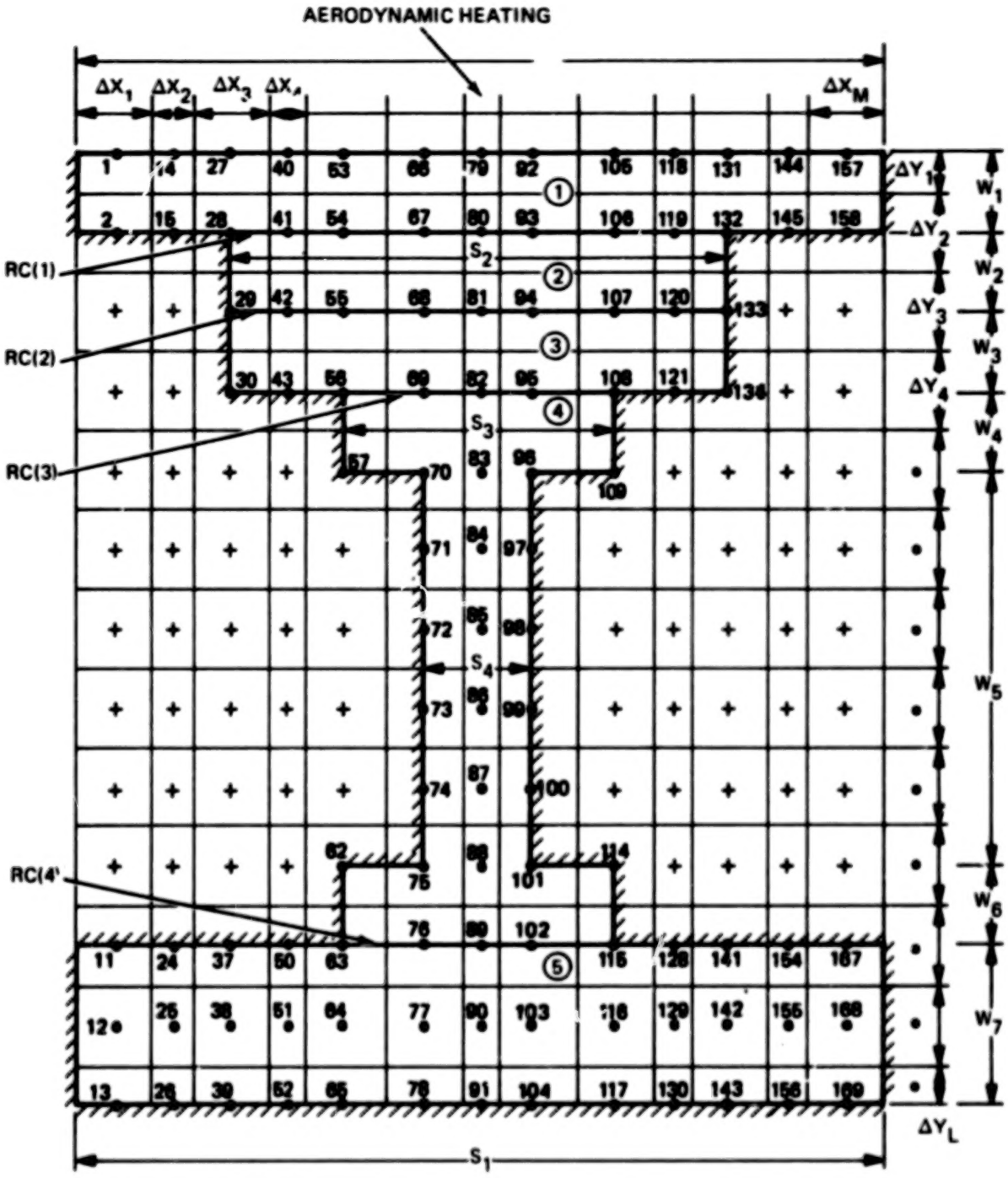


FIG. 11 GRID NETWORK FOR SLAB



FOR SAMPLE PROBLEM:
COMPONENT ① BERYLLIUM
COMPONENT ② INSULATION
COMPONENTS ③ , ④ AND ⑤ ALUMINUM

FIG. 12 GRID NETWORK FOR X-24C GEOMETRY

4.2 INPUT DATA FORMAT FOR COOLED PANEL AND SLAB GEOMETRIES

Indexes Card

- JGEO, L, M, NE* (4I5)

  JGEO = 0 (selects slab or cooled panel geometry)

  L = number of elements through the material

  M = number of elements along top

  NE = number of dominant eigenvalues to be used in solution (e.g., a typical number is 5)

Title Card

- Run identification, comments, etc. (5A10)

Radiation Card

- EPS1, TBG1 (2F10.5)

  EPS1 = emissivity of surface

  TBG1 = background radiation temperature, °R

Material Properties Cards

- MAT (I5)
- NMAT, RHO1, CONAV1, CPAV1 (I10, 3F10.5)
- TMAT1(1), TMAT1(2), ..., TMAT1(NMAT1) (8E10.0)
- CONDT1(1), CONDT1(2), ..., CONDT1(NMAT1) (8E10.0)
- CPT1(1), CPT1(2), ..., CPT1(NMAT1) (8E10.0)

omit if NMAT1=0

(If MAT = 2 include the cards:)

- NMAT2, RHO2, CONAV2, CPAV2 (I10, 3F10.5)
- TMAT2(1), TMAT2(2), ..., TMAT2(NMAT2) (8E10.0)
- CONDT2(1), CONDT2(2), ..., CONDT2(NMAT2) (8E10.0)
- CPT2(1), CPT2(2), ..., CPT2(NMAT2) (8E10.0)

omit if NMAT2=0

*Current dimension limitations require that the product L x M not exceed 200 and that M not exceed 50.

MAT = number of materials (1 or 2)

NMAT1 = number of entries in properties table (maximum of 10) NMAT1 = 0 for constant properties

RHO1 = density of material 1, lbm/cu-ft

CONAV1 = average thermal conductivity of material 1 (used when NMAT1 = 0), Btu/ft-sec-°R

CPAV1 = average specific heat of material 1 (used when NMAT1 = 0), Btu/lbm-°R

TMAT1(I) = temperatures in thermal properties table for which CONDT1(I) and CPT1(I) are given; I = 1, 2, ..., NMAT1, °R

CONDT1(I) = thermal conductivity of material 1 at temperature TMAT1(I), Btu/ft-sec-°R

CPT1(I) = specific heat of material 1 at temperature TMAT1(I), Btu/lbm-°R

NMAT2, RHO2, CONAV2, etc., same as NMAT1, RHO1, CONAV1, etc. except applied to material 2

Detail Geometry Cards

- DELX(1), DELX(2), DELX(3), ..., DELX(M) (8F10.5)
- DELY(1), DELY(2), DELY(3), ..., DELY(L) (8F10.5)
- TAU, R (2F10.5)
- S1, S2, W1, W2, W3, HEIGHT, TCOOL, HCOOL (8F10.5)
  (leave this card blank when considering a slab, Fig. 11)

DELX(I) = spatial increments in x direction I = 1, 2, ..., M, ft

DELY(I) = spatial increments in y direction I = 1, 2, ..., L, ft

TAU = thickness of material 1, ft

R = radius of inner corner (Fig. 8) (leave blank if it doesn't apply), ft (=0. for MAT=1)

S1 = (refer to Figs. 9 or 10), ft

S2 = (refer to Figs. 9 or 10), ft

W1 = (refer to Figs. 9 or 10), ft

W2 = (refer to Figs. 9 or 10), ft

W3 = (refer to Figs. 9 or 10), ft

HEIGHT = (refer to Figs. 9 or 10), ft

TCOOL = coolant temperature, °R

HCOOL = coolant heat transfer coefficient, Btu/ft^2-sec-°R

Initial Temperature Cards

- KODE, I, T(I), II, JJ (2I5, E10.0, 2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

T(I) = node initial temperature, °R

II and JJ = the node number is incremented by the spacing JJ until the limit II is reached. Each node so specified is assigned the same temperature

Boundary Condition Cards

Two options exist: (1) in the first option, the user inputs the flight trajectory and the code calculates the convective boundary conditions along the top surface of the panel in accordance with the equations presented in Appendix B; and (2) in the second option, the user inputs directly the convective heat transfer coefficient and adiabatic temperature as functions of time and distance.

OPTION 1. FLIGHT TRAJECTORY SPECIFIED

- REFX, CODEX, HMODI (3E10.0)
- GAM, RGAS, PR (3E10.0)
- NTRAJ (I10)
- TIMTAB(1), TIMTAB(2), ..., TIMTAB (NTRAJ) (8E10.0)
- ALTTAB(1), ALTTAB(2), ..., ALTTAB (NTRAJ) (8E10.0)
- VELTAB(1), VELTAB(2), ..., VELTAB (NTRAJ) (8E10.0)
- ALPTAB(1), ALPTAB(2), ..., ALPTAB (NTRAJ) (8E10.0)

REFX = effective boundary layer length, e.g., distance from leading edge or nose of vehicle (refer to Appendix B), ft

CODEX = 0. for uniform convective coefficient across top surface; CODEX = 1. for nonuniform convective coefficient (i.e., function of x); CODEX = -1. for tabular input for convective coefficient and adiabatic wall temperature

HMODI = nonzero value indicates that a table will be read at the end and used to multiply the convective coefficients

GAM = ratio of specific heats of air

RGAS = gas constant for air, ft-lbf/lbm-°R

PR = Prandtl number of air

NTRAJ = number of points in trajectory table ($2 \leq NTRAJ \leq 50$)

TIMTAB(I) = time in trajectory table I = 1, NTRAJ, sec

ALTTAB(I) = altitude corresponding to time TIMTAB(I), ft

VELTAB(I) = velocity corresponding to time TIMTAB(I), ft/sec

ALPTAB(I) = angle of attack corresponding to time TIMTAB(I), (must be non-negative), degrees

OPTION 2. CONVECTIVE COEFFICIENT AND ADIABATIC WALL TEMPERATURE SPECIFIED

- REFX, CODEX, HMODI (3E10.0)

REFX = effective boundary layer length, e.g., distance from leading edge of nose of vehicle (refer to Appendix B), ft

CODEX = -1. (indicates to code that Option 2 is being exercised)

HMODI = nonzero value indicates that a table will be read at the end and used to multiply the convective coefficient

Convective Coefficient and Adiabatic Wall Temperature Tables

Two tables are required. The first gives the convective coefficient as a function of time (argument 1) and distance (argument 2). The second table gives the adiabatic wall temperature as a function of time (argument 1) and distance (argument 2). In setting up these tables, the same values for time and distance must be used in both tables. The range of the distance argument must include the interval REFX to REFX plus S1 for the cooled panel geometry and REFX to REFX plus WIDTH, where WIDTH = $\sum_{i=1}^{M} \Delta X_i$ for the slab geometry. The tables must be followed by a blank card. The specifics on the format for the tables are given in the descriptions of subroutine NURED in Appendix D.

The following input data is required for both options:

Time Intervals Cards

- NTIMES (I10)
- TIMES(1), TIMES(2), ... TIMES (NTIMES) (8E10.0)

NTIMES(I) = number of points in time intervals array ($2 \leq$ NTIMES ≤ 50)
TIMES(1) = initial time (usually equals 0.) sec
TIMES(I) = time at which temperature is calculated and printed out I = 2, 3, . . . , NTIMES, sec

Convective Coefficient Modification Table (omit when HMODI = 0.)

One table is required to modify the convective coefficient on the aerodynamically heated surface. The multiplicative factor is given as a function of time (argument 1) and distance (argument 2). The writeup for subroutine NURED (Sheet D-1, Appendix D) gives the specifics on the format requirements. Follow this table with a blank card; omit the blank card if the table is not read in.

4.3 INPUT DATA FORMAT FOR X-24C GEOMETRY (See Fig. 12)

Indexes Card

- JGEO, L, M, NE* (4I5)

  JGEO = 2 (selects X-24C geometry)

  L = number of elements through the material

  M = number of elements along top

  NE = number of dominant eigenvalues to be used in solution (e.g., a typical number is 5)

Title Card

- Run identification, comments, etc. (5A10)

Radiation Card

- EPS1, TBG1 (2F10.5)

  EPS1 = emissivity of surface

  TBG1 = background radiation temperature, °R

Material Properties Cards

- MAT (I5)
- NMAT, RHO1, CONAV1, CPAV1 (I10, 3F10.5)
- TMAT1(1), TMAT1(2), ..., TMAT1(NMAT1) (8E10.0) } omit if NMAT1=0
- CONDT1(1), CONDT1(2), ..., CONDT1(NMAT1) (8E10.0) } omit if NMAT1=0
- CPT1(1), CPT1(2), ..., CPT1(NMAT1) (8E10.0) } omit if NMAT1=0

(If MAT = 2 include the cards:)

- NMAT2, RHO2, CONAV2, CPAV2 (I10.0, 3F10.5)
- TMAT2(1), TMAT2(2), ..., TMAT2(NMAT2) (8E10.0) } omit if NMAT2=0
- CONDT2(1), CONDT2(2), ..., CONDT2(NMAT2) (8E10.0) } omit if NMAT2=0
- CPT2(1), CPT2(2), ..., CPT2(NMAT2) (8E10.0) } omit if NMAT2=0

MAT = number of materials (1, 2 or 3)

NMAT1 = number of entries in properties table (maximum of 10) NMAT1 = 0 for constant properties

RHO1 = density of material 1, lbm/cu-ft

*Current dimension statements require L x M ≤ 200 and M ≤ 50.

CPAV1 = average specific heat of material 1 (used when NMAT1 = 0), Btu/lbm-°R

TMAT1(I) = temperatures in thermal properties table for which CONDT1(I) and CPT1(I) are given; I = 1, NMAT1, °R

CONDT1(I) = thermal conductivity of material 1 at temperature TMAT1(I), Btu/ft -sec-°R

CPT1(I) = specific heat of material 1 at temperature TMAT1(I), Btu/lbm-°R

NMAT2, RHO2, CONAV2, etc., same as NMAT1, RHO1, CONAV1, etc., except applied to material 2

Detail Geometry Cards

- DELX(1), DELX(2), DELX(3), ..., DELX(M) (8E10.0)
- DELY(1), DELY(2), DELY(3), ... DELY(L) (8E10.0)
- S1, S2, S3, S4 (4E10.0)
- W1, W2, W3, W4, W5, W6, W7 (7E10.0)
- RC(1), RC(2), RC(3), RC(4) (4E10.0)
- MP(1), MP(2), MP(3), MP(4), M(5) (5E10.0)

DELX(I) = spatial increments in x direction I = 1,2,...,M, ft

DELY(I) = spatial increments in y direction I = 1,2,...,L, ft

S1, S2, etc. = (Refer to Fig. 12) ft

RC(I) = unit surface contact resistance between components I and I+1, sec-sq-ft-°R/Btu

MP(I) = integer value 1, 2 or 3 to select material properties for component I

Initial Temperature Cards

- KODE, I, T(I), II, JJ
- ...
- ...
- ...
- 11100

KODE = 0 or blank

I = node number

T(I) = node initial temperature, °R

II and JJ = the node number is incremented by the spacing JJ until the limited II is reached. Each node so specified is assigned the same temperature

Boundary Condition Cards

Two options exist: (1) in the first option, the user inputs the flight trajectory and the code calculates the convective boundary conditions along the top surface of the panel in accordance with the equations presented in Appendix B; and (2) in the second option, the user inputs directly the convective heat transfer coefficient and adiabatic temperature as functions of time and distance.

OPTION 1. FLIGHT TRAJECTORY SPECIFIED

- REFX, CODEX, HMODI (3E10.0)
- GAM, RGAS, PR (3E10.0)
- NTRAJ (I10)
- TIMTAB(1), TIMTAB(2), ..., TIMTAB (NTRAJ) (8E10.0)
- ALTTAB(1), ALTTAB(2), ..., ALTTAB (NTRAJ) (8E10.0)
- VELTAB(1), VELTAB(2), ..., VELTAB (NTRAJ) (8E10.0)
- ALPTAB(1), ALPTAB(2), ..., ALPTAB (NTRAJ) (8E10.0)

REFX = effective boundary layer length, e.g., distance from leading edge or nose of vehicle (refer to Appendix B), ft

CODEX = 0. for uniform convective coefficient across top surface; CODEX = 1. for nonuniform convective coefficient (i.e., function of x); CODEX = -1. for tabular input for convective coefficient and adiabatic wall temperature

HMODI = nonzero value indicates that a table will be read at the end and used to multiply the convective coefficients

GAM = ratio of specific heats of air

RGAS = gas constant for air, ft-lbf/lbm-°R

PR = Prandtl number of air

NTRAJ = number of points in trajectory table ($2 \leq NTRAJ \leq 50$)

TIMTAB(I) = time in trajectory table I = 1, NTRAJ, sec

ALTTAB(I) = altitude corresponding to time TIMTAB(I), ft

VELTAB(I) = velocity corresponding to time TIMTAB(I), ft/sec

ALPTAB(I) = angle of attack corresponding to time TIMTAB(I), in degrees

OPTION 2. CONVECTIVE COEFFICIENT AND ADIABATIC WALL TEMPERATURE SPECIFIED

- REFX, CODEX, HMODI (3E10.0)

REFX = effective boundary layer length, e.g., distance from leading edge or nose of vehicle (refer to Appendix B), ft

CODEX = -1. (indicates to code that Option 2 is being exercised)

HMODI = nonzero value indicates that a table will be read at the end and used to multiply the convective coefficients

Convective Coefficient and Adiabatic Wall Temperature Tables

Two tables are required. The first gives the convective coefficient as a function of time (argument 1) and distance (argument 2). The second table gives the adiabatic wall temperature as a function of time (argument 1) and distance (argument 2). In setting up these tables, the same values for time and distance must be used in both tables. The range of the distance argument must include the interval REFX to REFX plus S1 for the cooled panel geometry and REFX to REFX plus WIDTH, where WIDTH = $\sum_{i=1}^{M} \Delta X_i$ for the slab geometry. The tables must be followed by a blank card. The specifics on the format for the tables are given in the descriptions of subroutine NURED in Appendix D.

The following input data is required for both options:

Time Intervals Cards

- NTIMES (I10)
- TIMES(1), TIMES(2), ... TIMES (NTIMES) (8E10.0)

NTIMES(I) = number of points in time intervals array ($2 \leq NTIMES \leq 50$)

TIMES(1) = initial time (usually equals 0.) sec

TIMES(I) = time at which temperature is calculated and printed out I = 2,3,...,NTIMES, sec

Convective Coefficient Modification Table (omit when HMODI = 0.)

One table is required to modify the convective coefficient on the aerodynamically heated surface. The multiplicative factor is given as a function of time (argument 1) and distance (argument 2). The writeup for subroutine NURED (Sheet D-1, Appendix D) gives the specifics on the format requirements. Follow this table with a blank card; omit the blank card if the table isn't read in.

4.4 SAMPLE PROBLEM FOR COOLED PANEL GEOMETRY

This subsection contains an illustration of the cooled panel geometry considered as a sample problem (see Sheet 4.4). Figures 9 and 10 show the grid network. The panel was constructed of aluminum with a 0.686 cm layer of beryllium applied across the top in contact with the hot boundary layer air. The aluminum structure was cooled by a coolant at a temperature of 660°R with a convective heat transfer coefficient of 612.8 watts/m^2K.

Aerodynamic heating was calculated by CAVE based on the missile flight trajectory shown in the following listing of the input data. The panel was located on the missile 6.096 m aft of the start of the boundary layer and hence the input variable REFX equals 20. Throughout the flight trajectory the angle of attack was 20 degrees.

This sample problem shows the use of the heating multiplier option. Examination of the heating multiplier table in the listing of the input data reveals that during the time period 30 to 40.4 seconds, the convective heat transfer coefficient increases fivefold for the center section of the panel. To represent this step change in the heating, table entries of 29.9, 30.1, 40.3 and 40.5 have been used. Furthermore the time step intervals array contains these same times. Whenever a step change is to be approximated to CAVE, it is necessary to use "bracketing" times in setting up a multiplier table and the time step interval array. That is, if a step change occurs at time t_o, the table should have time entries at $t_o - \epsilon$ and $t_o + \epsilon$ where ϵ is some small number. Likewise the time step interval array should have these same times.

The following pages show listings of the input data and the output generated by CAVE for this cooled panel problem. The sequence of the output is:

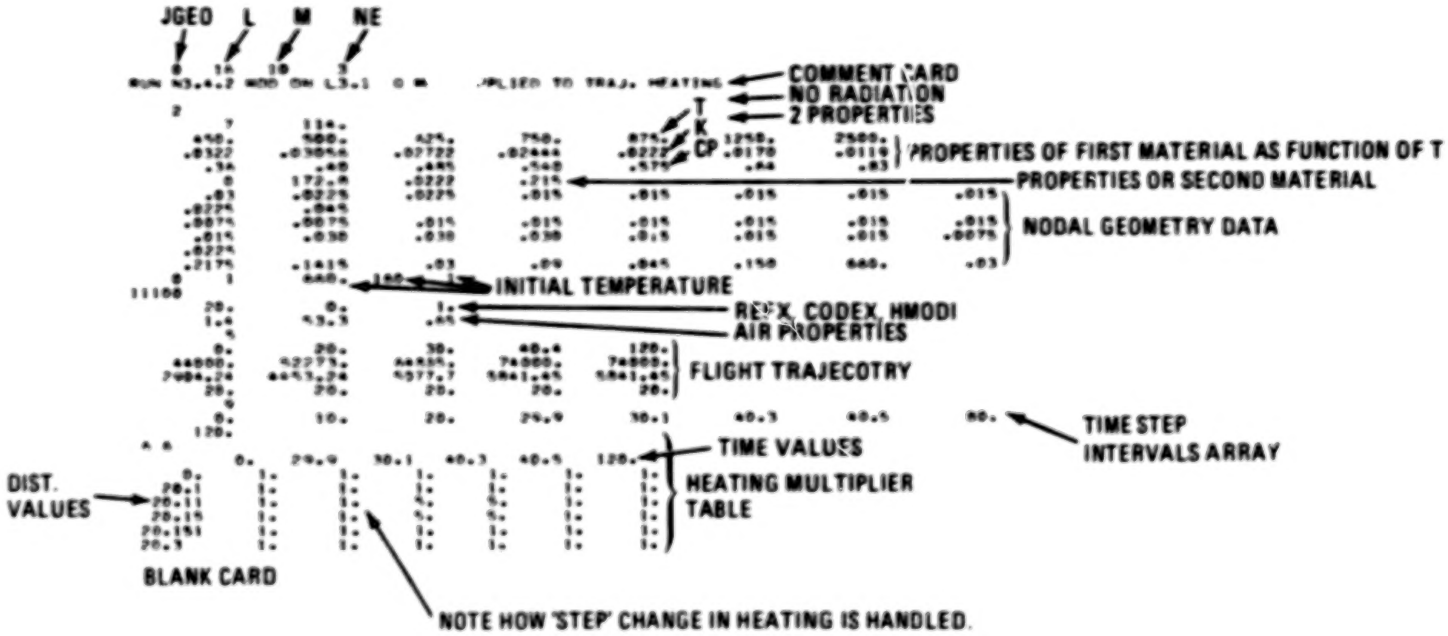
- Statement regarding storage allocated for S array in main program
- Geometry related input data
- Material properties illustrating the format used when the properties are temperature dependent and temperature independent
- Flight trajectory table
- Table of heating multiplier as a function of time and distance

- Node number location within the output array. A node number of 0 (zero) printed in this array indicates a nonactive node. Notice how the printout has a format similar to the shape of the geometry of the problem being considered, which makes for very convenient reading and quickly gives the user a feel for the temperature distribution within the body. This format is carried throughout in all the following arrays
- Material number assigned to each node. A material number of 3 signifies that the node is at an interface between material 1 and 2
- Capacitance of each node
- Conductance in the x-direction between nodes
- Conductance in the y-direction between nodes
- Initial temperature distribution

And then, the following information is printed for each time interval:

- Flight trajectory parameters, Mach number, altitude, velocity, and angle of attack at the end of the interval
- Average heat transfer coefficients calculated using h values at the beginning and end of this time interval
- Average heat transfer couplings, which include radiation effects, if any, calculated using the temperatures at the beginning of this time interval (See Appendix C)
- Average adiabatic wall temperatures for this time interval
- Temperatures at the end of the interval
- Steady-state temperatures for the heat transfer couplings and adiabatic wall temperatures of this interval
- Integrated heat input to each node. This gives the net heat transfer at each boundary node up to the end of this time interval. A positive value signifies heat transfer into the body and a negative value heat transfer out of the body at that boundary node

(To aid the reader, annotation has been added to the listing of the input data and to the output.)



SHEET 4.4 INPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 1 OF 8)

NO. OF ELEMENTS ALONG TOP SURFACE
L
M
MAX. NODE NO.
NO. OF EIGENVALUES NE
CAVE CODE
10 SURFACE ELEMENTS-- 16 ROWS BY 10 COLUMNS GIVES 160 ELEMENTS 3 DOMINANT MODES...REQUIRES 5357 WORDS OF MEMORY
ECONOMIZE...REDUCE DIMENSION OF 5 AND VALUE OF NWORDS FROM 12000 TOWARDS 6108
VALUE REQUIRED FOR THIS PROBLEM
VALUE REQUESTED IN CAVE
RUN NS.4.2 MOD ON L3.1 0 MULT APPLIED TO TRAJ. HEAT
COMMENT CARD

C O O L E D P A N E L P R O B L E M

S1= 0.2175E 00 FT
S2= 0.1415E 00 FT
TAU= 0.2250E-01 FT
EMISSIVITY= 0.0
RADIATION BACKGROUND T= 0.0 DEG.R
W1= 0.3000E-01 FT
W2= 0.9000E-01 FT
W3= 0.4500E-01 FT
HT= 0.1500E 00 FT
COOLANT HH= 0.3000E-01 BTU/SEC-FT**2-DEG.R
COOLANT T= 0.6600E 03 DEG.R

PRINTOUT OF INPUT DATA

I	DELX(I)									
1	0.3000E-01	2	0.2250E-01	3	0.2250E-01	4	0.1500E-01	5	0.1500E-01
6	0.1500E-01	7	0.1500E-01	8	0.1500E-01	9	0.2250E-01	10	0.4500E-01

I	DELY(I)									
1	0.7500E-02	2	0.7500E-02	3	0.1500E-01	4	0.1500E-01	5	0.1500E-01
6	0.1500E-01	7	0.1500E-01	8	0.1500E-01	9	0.1500E-01	10	0.3000E-01
11	0.3000E-01	12	0.3000E-01	13	0.1500E-01	14	0.1500E-01	15	0.1500E-01
16	0.7500E-02								

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 2 OF 8)

MATERIAL PROPERTIES

MATERIAL 1	RHO=114.00 LBM/CU-FT	K=0.03220 BTU/SEC-FT-DEG.R	CP=0.3600 BTU/LBM-DEG.R	T= 450.00 DEG.R
		K=0.03096 BTU/SEC-FT-DEG.R	CP=0.4000 BTU/LBM-DEG.R	T= 500.00 DEG.R
		K=0.02772 BTU/SEC-FT-DEG.R	CP=0.4850 BTU/LBM-DEG.R	T= 625.00 DEG.R
		K=0.02444 BTU/SEC-FT-DEG.R	CP=0.5450 BTU/LBM-DEG.R	T= 750.00 DEG.R
		K=0.02220 BTU/SEC-FT-DEG.R	CP=0.5740 BTU/LBM-DEG.R	T= 875.00 DEG.R
		K=0.01750 BTU/SEC-FT-DEG.R	CP=0.6400 BTU/LBM-DEG.R	T=1250.00 DEG.R
		K=0.01190 BTU/SEC-FT-DEG.R	CP=0.8300 BTU/LBM-DEG.R	T=2400.00 DEG.R
MATERIAL 2	RHO=172.80 LBM/CU-FT	K=0.02220 BTU/SEC-FT-DEG.R	CP=0.2150 BTU/LBM-DEG.R	

1ST MATERIAL HAS TEMPERATURE DEPENDENT PROPERTIES

2ND MATERIAL

TABLES

TIME IN SECONDS				
0.0	0.2000E 02	0.3000E 02	0.4040E 02	0.1200E 03
ALTITUDE IN FEET				
0.4400E 04	0.4927E 04	0.4487E 04	0.7400E 04	0.7400E 04
VELOCITY IN FEET PER SEC.				
0.2904E 04	0.4435E 04	0.5874E 04	0.5841E 04	0.5841E 04
ANGLE OF ATTACK IN DEGREES				
0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02

FLIGHT TRAJECTORY TABLE

TABLE OF HEATING MULTIPLIER AS A FUNCTION OF TIME AND DISTANCE

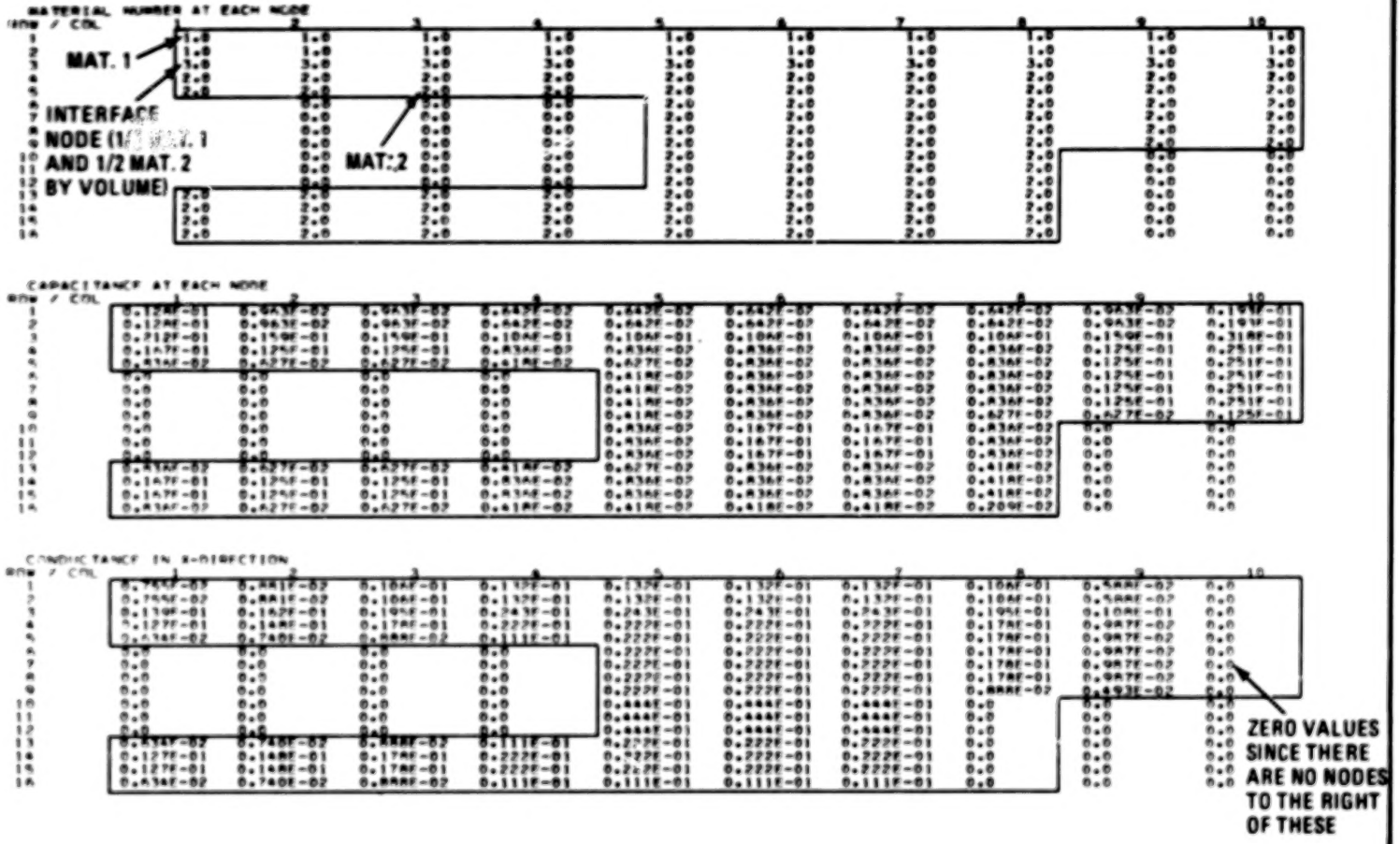
A	A								
0.0	0.0	0.2998E 02	0.3010E 02	0.4038E 02	0.4050E 02	0.1200E 03	0.0	0.0	0.0
0.0	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
0.2810E 02	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
0.2811E 02	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
0.2815E 02	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
0.2816E 02	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
0.2830E 02	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.0	0.0	0.0
B	B								

NODAL NETWORK

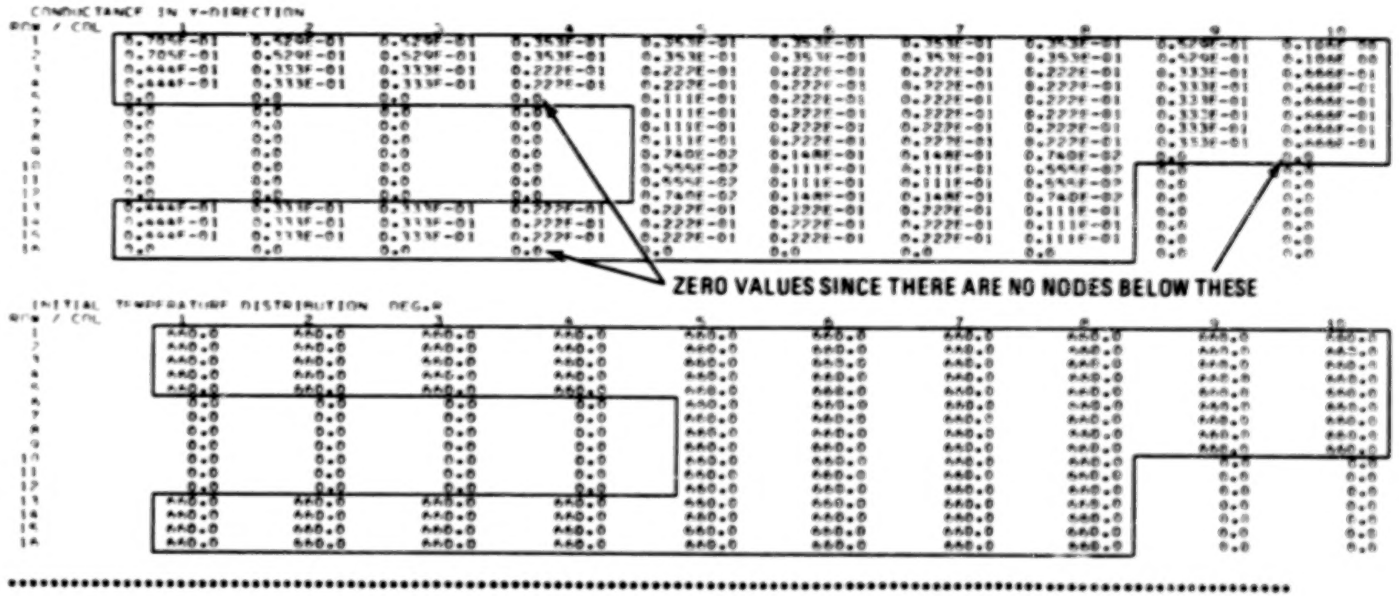
PRINTOUT HAS FORMAT SIMILAR TO SHAPE OF GEOMETRY

NODE NUMBERS ROW / COL	1	2	3	4	5	6	7	8	9	10
1	1.0	17.0	33.0	49.0	65.0	81.0	97.0	113.0	129.0	145.0
2	2.0	18.0	34.0	50.0	66.0	82.0	98.0	114.0	130.0	146.0
3	3.0	19.0	35.0	51.0	67.0	83.0	99.0	115.0	131.0	147.0
4	4.0	20.0	36.0	52.0	68.0	84.0	100.0	116.0	132.0	148.0
5	5.0	21.0	37.0	53.0	69.0	85.0	101.0	117.0	133.0	149.0
6	0.0	0.0	0.0	0.0	70.0	86.0	102.0	118.0	134.0	150.0
7	0.0	0.0	0.0	0.0	71.0	87.0	103.0	119.0	135.0	151.0
8	0.0	0.0	0.0	0.0	72.0	88.0	104.0	120.0	136.0	152.0
9	0.0	0.0	0.0	0.0	73.0	89.0	105.0	121.0	137.0	153.0
10	0.0	0.0	0.0	0.0	74.0	90.0	106.0	122.0	0.0	0.0
11	0.0	0.0	0.0	0.0	75.0	91.0	107.0	123.0	0.0	0.0
12	0.0	0.0	0.0	0.0	76.0	92.0	108.0	124.0	0.0	0.0
13	13.0	29.0	45.0	61.0	77.0	93.0	109.0	125.0	0.0	0.0
14	14.0	30.0	46.0	62.0	78.0	94.0	110.0	126.0	0.0	0.0
15	15.0	31.0	47.0	63.0	79.0	95.0	111.0	127.0	0.0	0.0
16	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	0.0	0.0

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 3 OF 8)

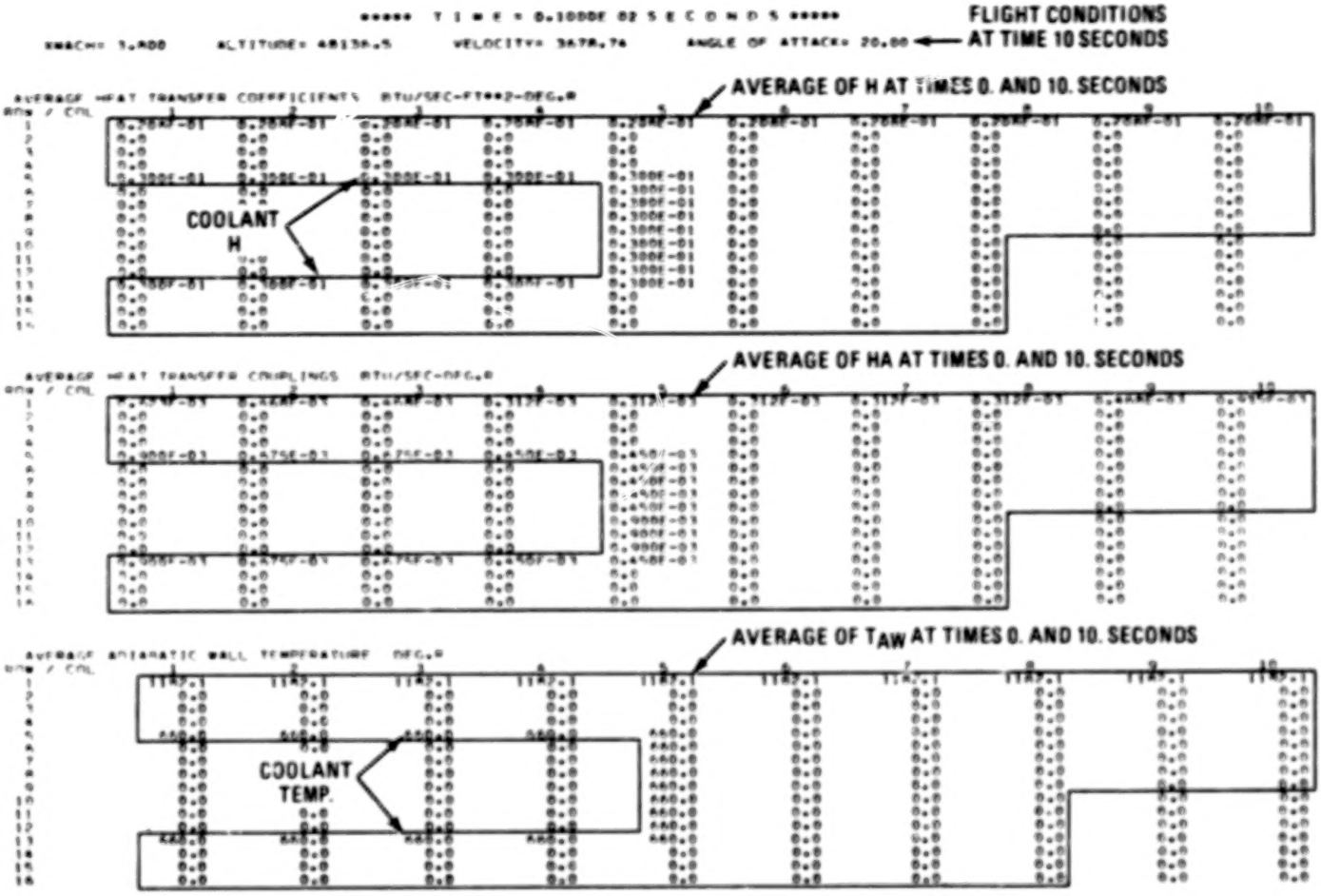


SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 4 OF 8)


CONDUCTANCE IN Y-DIRECTION
ROW / COL
ZERO VALUES SINCE THERE ARE NO NODES BELOW THESE
INITIAL TEMPERATURE DISTRIBUTION DEG.R
ROW / COL

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 5 OF 8)

C01


***** T I M E = 0.1000E 02 S E C O N D S *****
FLIGHT CONDITIONS AT TIME 10 SECONDS
AVERAGE OF H AT TIMES 0. AND 10. SECONDS
COOLANT H
AVERAGE OF HA AT TIMES 0. AND 10. SECONDS
AVERAGE OF T_{AW} AT TIMES 0. AND 10. SECONDS
COOLANT TEMP.

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 6 OF 8)

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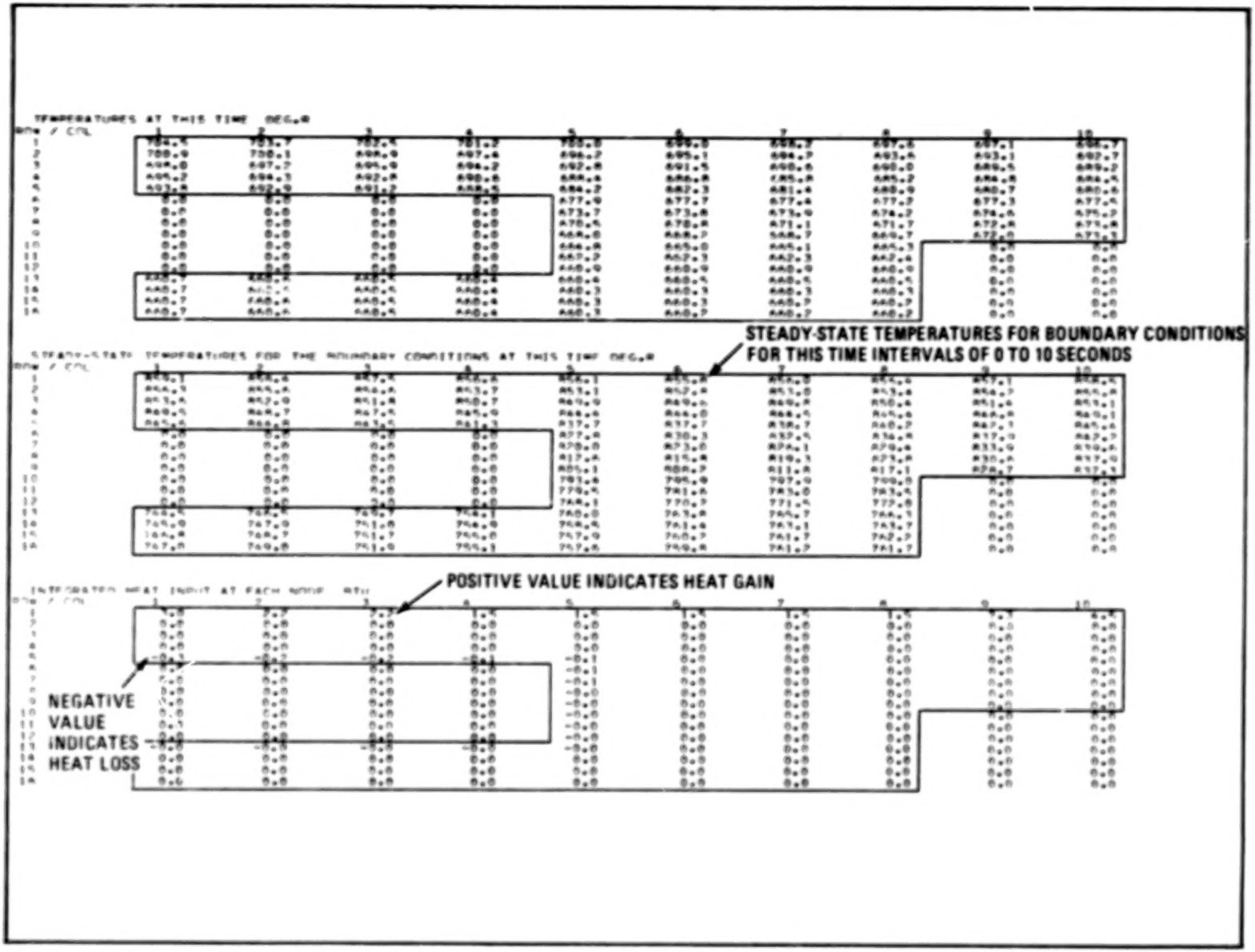
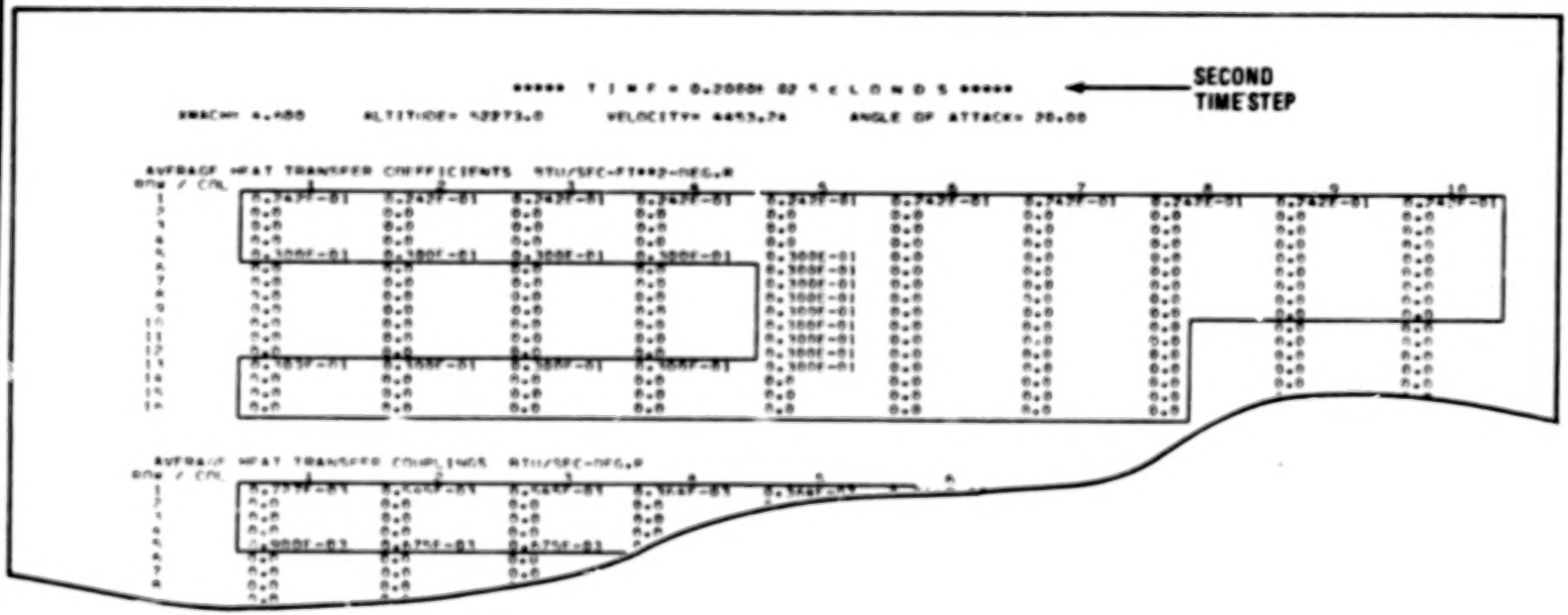

STEADY-STATE TEMPERATURES FOR BOUNDARY CONDITIONS FOR THIS TIME INTERVALS OF 0 TO 10 SECONDS
POSITIVE VALUE INDICATES HEAT GAIN
NEGATIVE VALUE INDICATES HEAT LOSS

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10) (SHEET 7 OF 8)


***** T I M E = 0.20000 02 S E C O N D S *****
SECOND TIME STEP
XMACH= 4.400 ALTITUDE= 42273.0 VELOCITY= 4493.74 ANGLE OF ATTACK= 20.00
AVERAGE HEAT TRANSFER COEFFICIENTS BTU/SEC-FT**2-DEG.R
0.7429E-01
0.3809E-01
0.1868E-01

SHEET 4.4 OUTPUT DATA FOR COOLED PANEL PROBLEM (REFER TO FIGS. 9 & 10)(SHEET 8 OF 8)

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4.5 SAMPLE PROBLEM FOR SLAB GEOMETRY

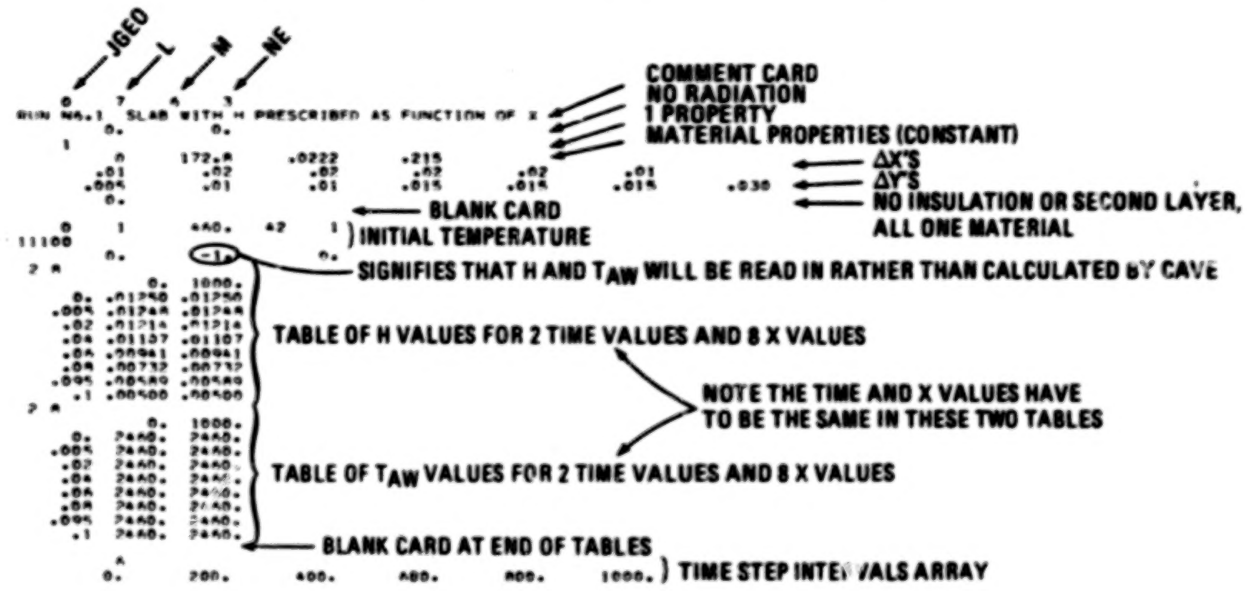
Figure 11 illustrates the grid network for the slab geometry considered as a sample problem. Aluminum was used throughout the slab and hence TAU = 0.

This sample problem illustrates the use of the option to input the convective heat transfer coefficients, h, and adiabatic wall temperatures, T_{AW}, as functions of time and distance along the top surface rather than have CAVE calculate them based on a flight trajectory. The values for h that were input were based on the equation

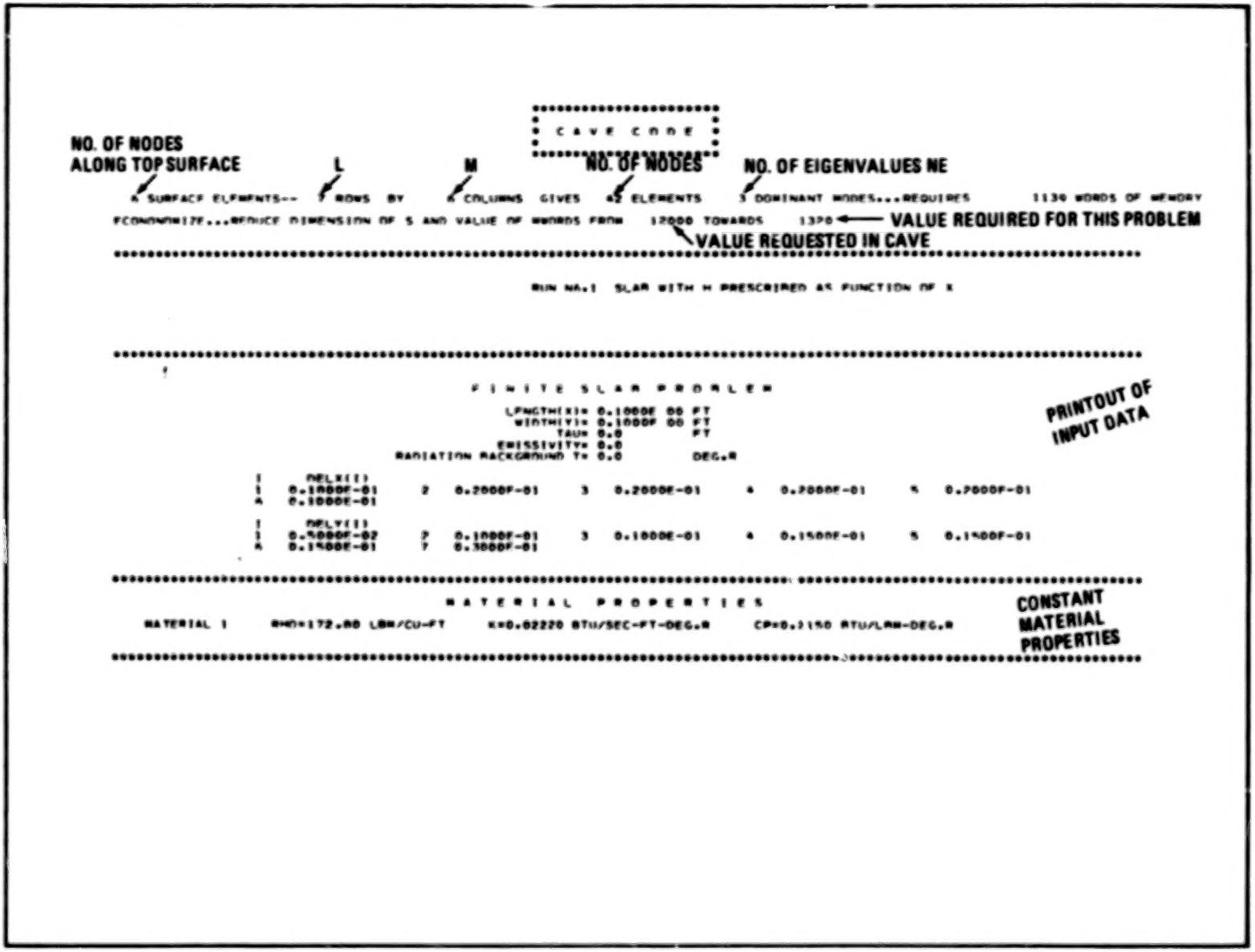
$$h = 0.005 + 0.0075 \cos(5 \pi x)$$

The adiabatic wall temperature was taken to be constant with respect to both time and distance.

Since the slab is rectangular in shape, the output is particularly easy to read; a row in an output array giving the values associated with a row of nodes. The type of output is the same as was described in detail for the cooled panel geometry (see subsection 4.4).


JGEO
L
M
NE
COMMENT CARD
NO RADIATION
1 PROPERTY
MATERIAL PROPERTIES (CONSTANT)
ΔX'S
ΔY'S
NO INSULATION OR SECOND LAYER, ALL ONE MATERIAL
BLANK CARD
INITIAL TEMPERATURE
SIGNIFIES THAT H AND T_{AW} WILL BE READ IN RATHER THAN CALCULATED BY CAVE
TABLE OF H VALUES FOR 2 TIME VALUES AND 8 X VALUES
NOTE THE TIME AND X VALUES HAVE TO BE THE SAME IN THESE TWO TABLES
TABLE OF T_{AW} VALUES FOR 2 TIME VALUES AND 8 X VALUES
BLANK CARD AT END OF TABLES
TIME STEP INTERVALS ARRAY

SHEET 4.5 INPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11)(SHEET 1 OF 6)


CAVE CODE
NO. OF NODES ALONG TOP SURFACE
L
M
NO. OF NODES
NO. OF EIGENVALUES NE
4 SURFACE ELEMENTS-- 7 ROWS BY 4 COLUMNS GIVES 42 ELEMENTS 3 DOMINANT MODES...REQUIRES 1130 WORDS OF MEMORY
RECOMMENDED...REDUCE DIMENSION OF 5 AND VALUE OF WORDS FROM 12000 TOWARDS 1370
VALUE REQUIRED FOR THIS PROBLEM
VALUE REQUESTED IN CAVE
RUN NO.1 SLAB WITH H PRESCRIBED AS FUNCTION OF X
FINITE SLAB PROBLEM
LENGTH(X)= 0.1000E 00 FT
WIDTH(Y)= 0.1000E 00 FT
TAU= 0.0 FT
EMISSIVITY= 0.0
RADIATION BACKGROUND T= 0.0 DEG.R
PRINTOUT OF INPUT DATA
I DELX(I)
1 0.1000E-01 2 0.2000E-01 3 0.2000E-01 4 0.2000E-01 5 0.2000E-01
6 0.1000E-01
I DELY(I)
1 0.5000E-02 2 0.1000E-01 3 0.1000E-01 4 0.1500E-01 5 0.1500E-01
6 0.1500E-01 7 0.5000E-01
MATERIAL PROPERTIES
MATERIAL 1 RHO=172.80 LBM/CU-FT K=0.02220 BTU/SEC-FT-DEG.R CP=0.3150 BTU/LBM-DEG.R
CONSTANT MATERIAL PROPERTIES

SHEET 4.5 OUTPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11) (SHEET 2 OF 6)

TABLES READ IN FOR H AND T_{AW}

TABLE
H AND TAW AS FUNCTIONS OF TIME AND DISTANCE

2	A										
0.0		0.0	0.1000F 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0		0.1250F-01	0.1250E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5000F-02		0.1248F-01	0.1248E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2000F-01		0.1214F-01	0.1214E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4000F-01		0.1107F-01	0.1107E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8000F-01		0.9418F-02	0.9418E-02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8000F-01		0.7320F-02	0.7320E-02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9500F-01		0.5869F-02	0.5869E-02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000F 00		0.5000F-02	0.5000E-02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	A										
0.0		0.0	0.1000E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5000F-02		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2000F-01		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4000F-01		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8000F-01		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.8000F-01		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.9500F-01		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000F 00		0.24A0F 04	0.2460E 04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0	0										

NODAL NETWORK

HEATING ON THIS FACE

NODE NUMBERS

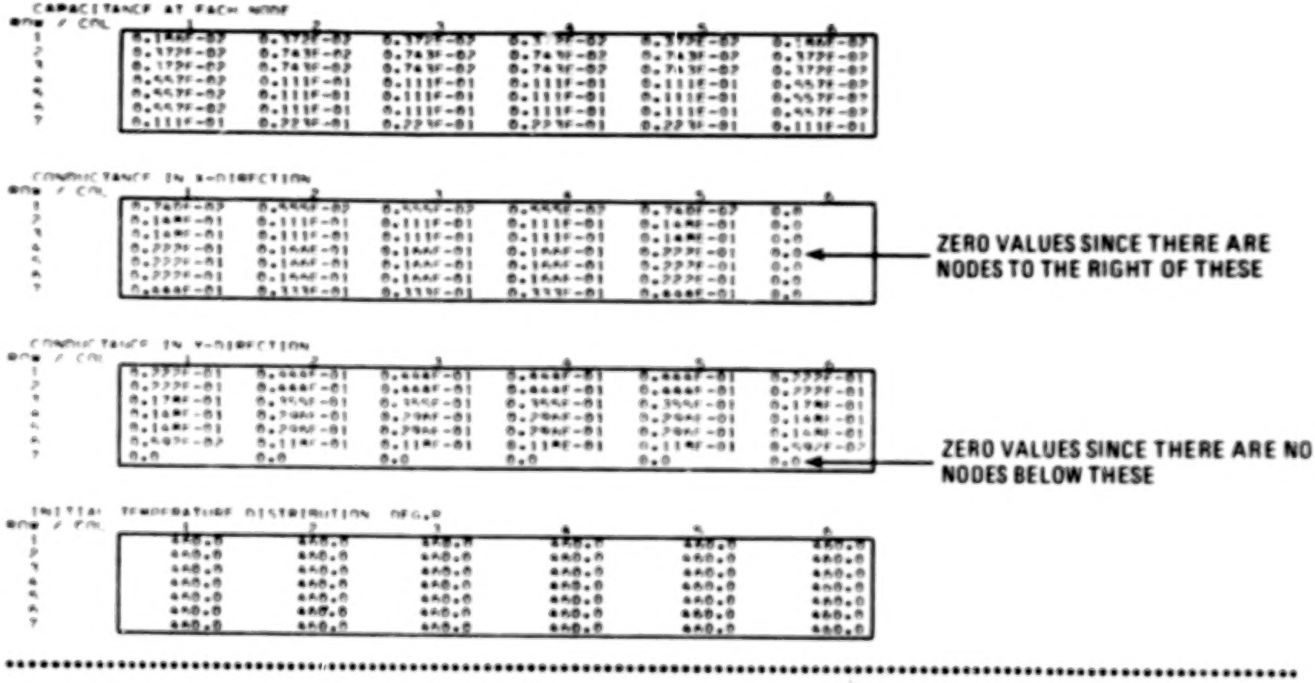
ROW / COL	1	2	3	4	5	6
1	1.0	8.0	15.0	22.0	29.0	36.0
2	2.0	9.0	16.0	23.0	30.0	37.0
3	3.0	10.0	17.0	24.0	31.0	38.0
4	4.0	11.0	18.0	25.0	32.0	39.0
5	5.0	12.0	19.0	26.0	33.0	40.0
6	6.0	13.0	20.0	27.0	34.0	41.0
7	7.0	14.0	21.0	28.0	35.0	42.0

MATERIAL NUMBER AT EACH NODE

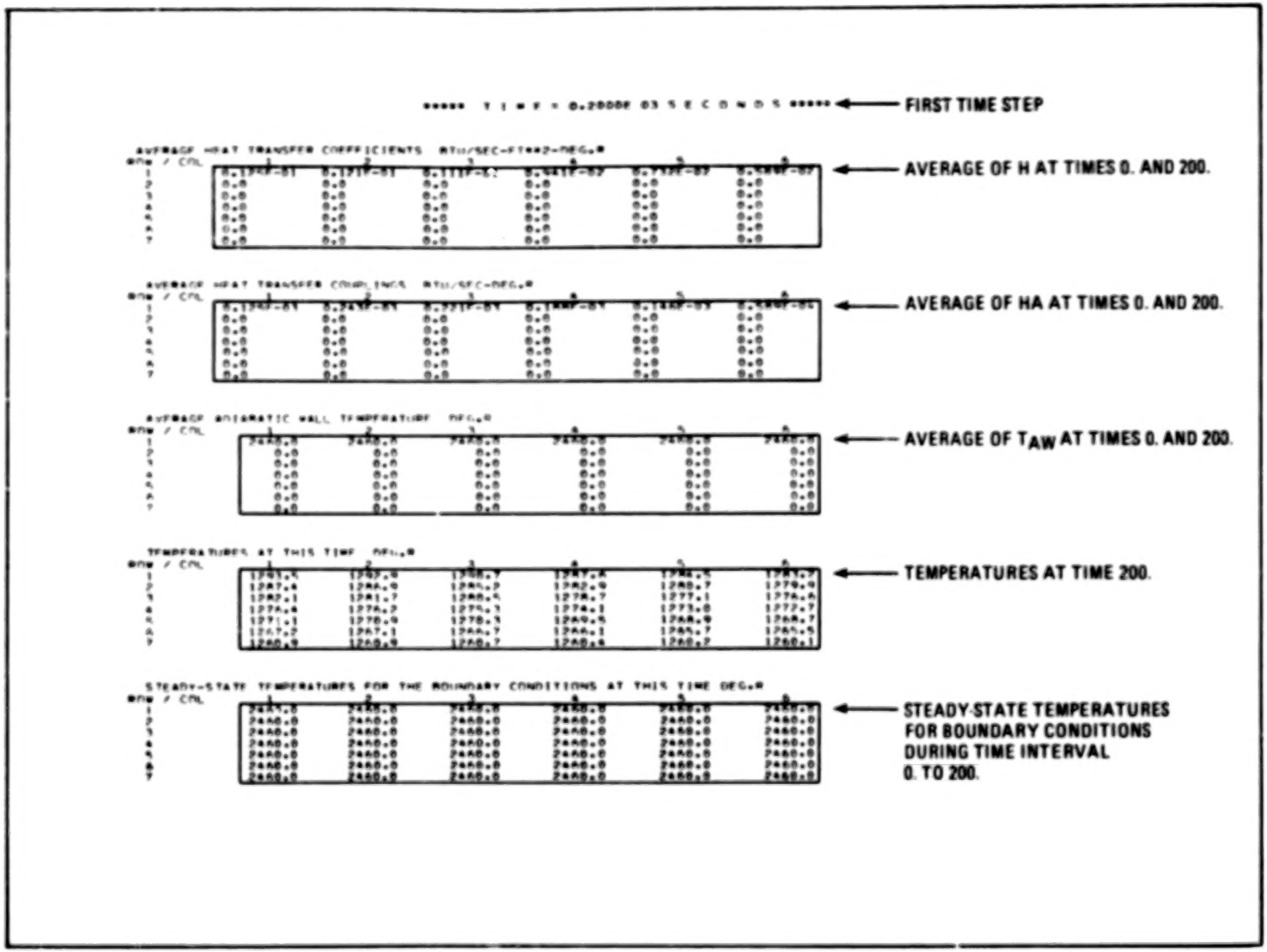
ROW / COL	1	2	3	4	5	6
1	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	1.0
4	1.0	1.0	1.0	1.0	1.0	1.0
5	1.0	1.0	1.0	1.0	1.0	1.0
6	1.0	1.0	1.0	1.0	1.0	1.0
7	1.0	1.0	1.0	1.0	1.0	1.0

ONE MATERIAL THROUGHOUT

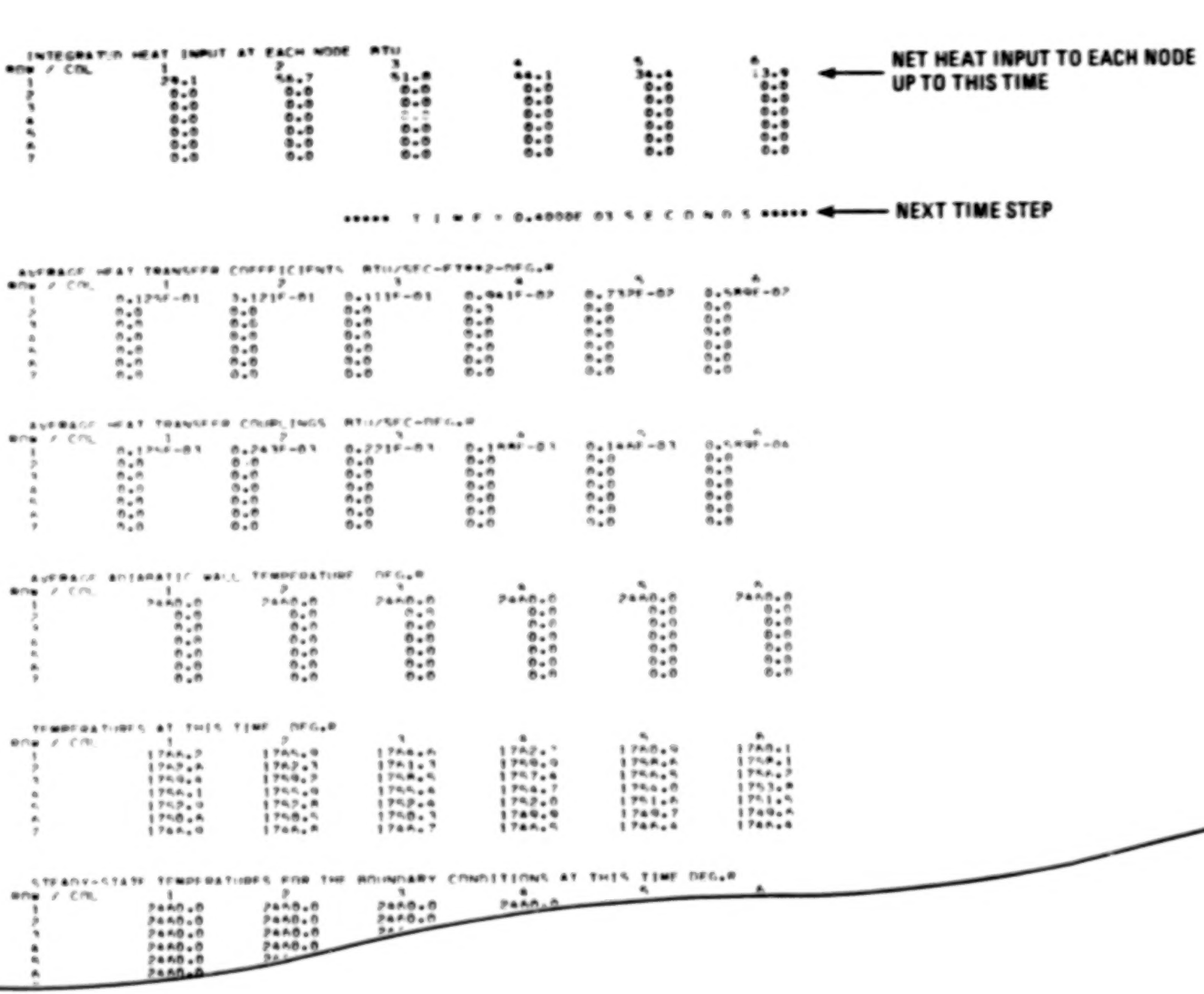
SHEET 4.5 OUTPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11) (SHEET 3 OF 6)


CAPACITANCE AT EACH NODE
ZERO VALUES SINCE THERE ARE NODES TO THE RIGHT OF THESE
ZERO VALUES SINCE THERE ARE NO NODES BELOW THESE

SHEET 4.5 OUTPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11) (SHEET 4 OF 6)


***** TIME = 0.20000 03 SECONDS *****
FIRST TIME STEP
AVERAGE HEAT TRANSFER COEFFICIENTS BTU/SEC-FT**2-DEG.R
AVERAGE OF H AT TIMES 0. AND 200.
AVERAGE HEAT TRANSFER CONDUCTANCE BTU/SEC-DEG.R
AVERAGE OF HA AT TIMES 0. AND 200.
AVERAGE ADIABATIC WALL TEMPERATURE DEG.R
AVERAGE OF T_{AW} AT TIMES 0. AND 200.
TEMPERATURES AT THIS TIME DEG.R
TEMPERATURES AT TIME 200.
STEADY-STATE TEMPERATURES FOR THE BOUNDARY CONDITIONS AT THIS TIME DEG.R
STEADY-STATE TEMPERATURES FOR BOUNDARY CONDITIONS DURING TIME INTERVAL 0. TO 200.

SHEET 4.5 OUTPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11) (SHEET 5 OF 6)



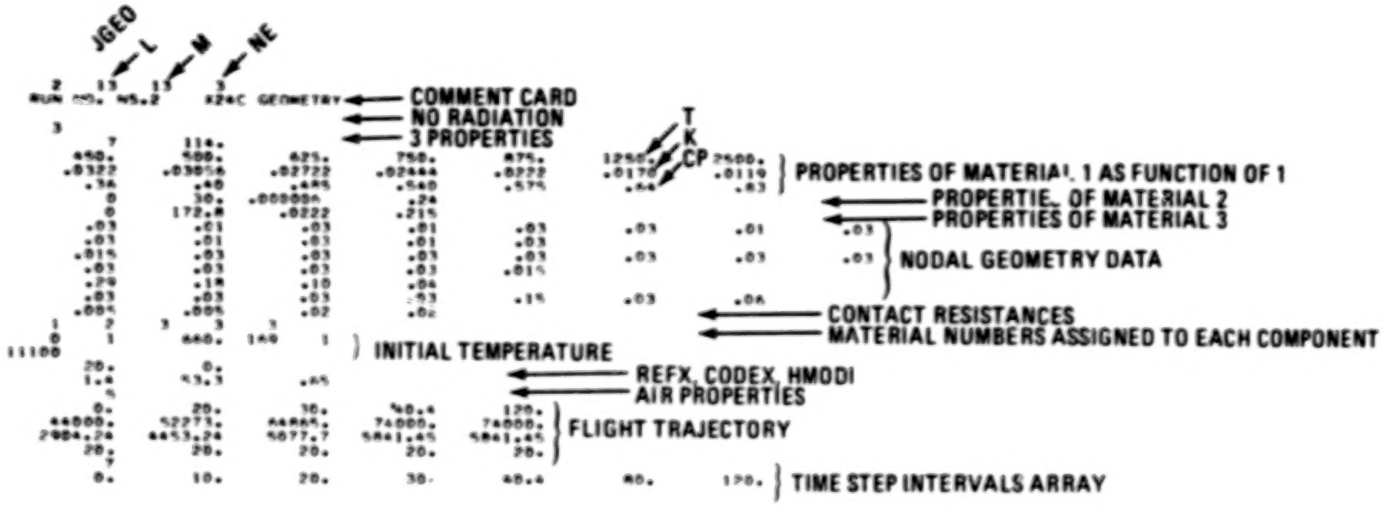
SHEET 4.5 OUTPUT DATA FOR SLAB PROBLEM (REFER TO FIG. 11) (SHEET 6 OF 6)

4.6 SAMPLE PROBLEM X-24C GEOMETRY

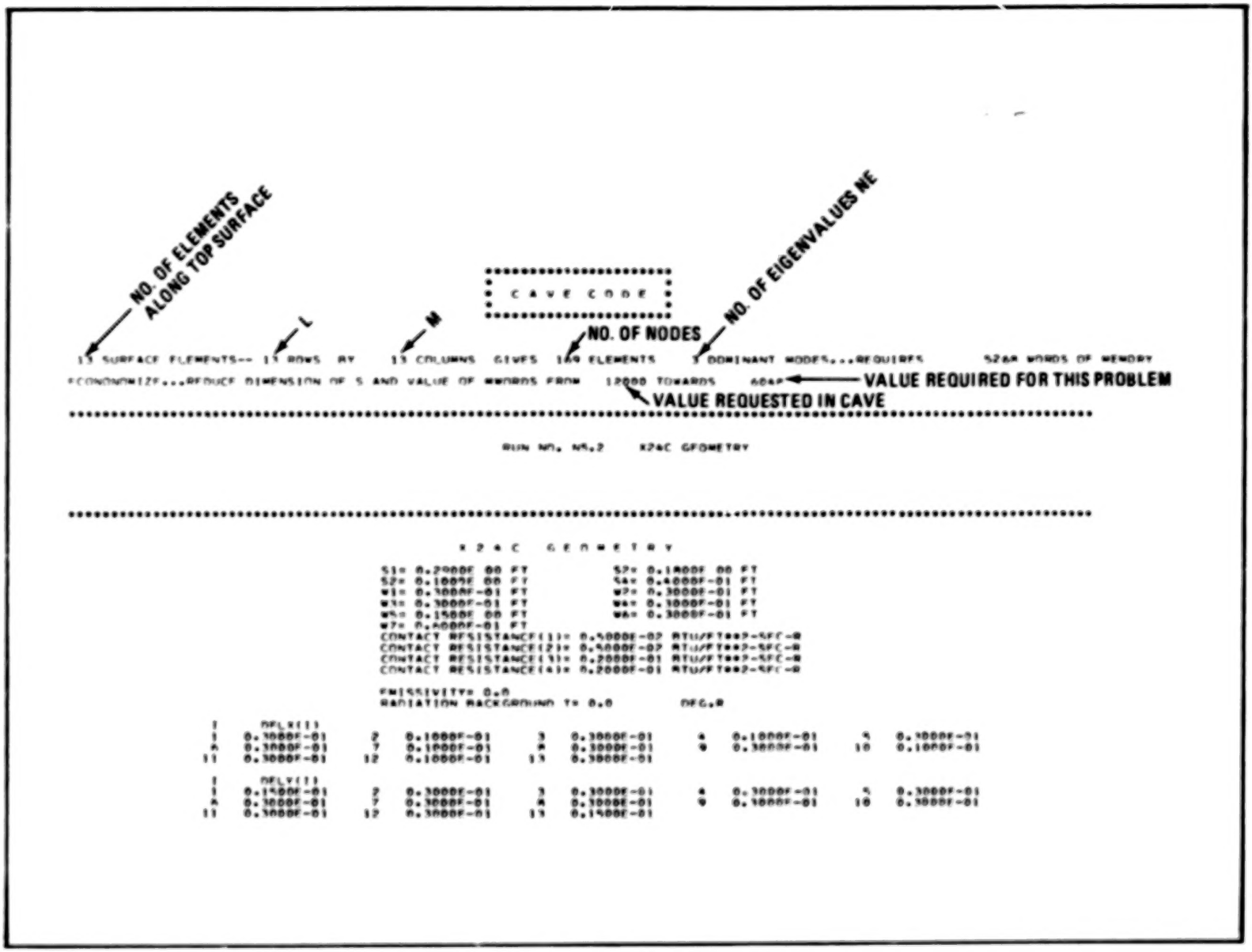
Figure 12 shows the grid network for the X-24C geometry considered as a sample problem.

In many respects the input and output for this problem are similar to that of the cooled panel problem presented in subsection 4.4. One of the differences is that there are three materials here instead of two which affects the array of material numbers.

For the X-24C geometry a material number of 1, 2 or 3 at a node signifies that a node is comprised entirely of material 1, 2 or 3. (Recall that the properties of the materials is established via the input.) And a two-digit material number signifies that the node is at an interface between two materials, with the tens digit giving the material number for the upper material and the ones digit the material number for the lower material at the interface. For example, node 54 is located at the interface between the beryllium and the insulation, since beryllium has been set up to be material 1 and the insulation to be material 2 we find the material number of node 54 to be 12. Material numbers are set up by subroutine X-24C and they provide a check on whether geometry data has been input correctly to CAVE. (The input and output listings on the following pages have been annotated.)



SHEET 4.6 INPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 1 OF 9)


NO. OF ELEMENTS ALONG TOP SURFACE
L
M
NO. OF NODES
NO. OF EIGENVALUES NE
CAVE CODE
15 SURFACE ELEMENTS-- 15 ROWS BY 15 COLUMNS GIVES 169 ELEMENTS 3 DOMINANT MODES...REQUIRES 5268 WORDS OF MEMORY
*CONDUM12P...REDUCE DIMENSION OF S AND VALUE OF MWORDS FROM 12000 TOWARDS 6048
VALUE REQUIRED FOR THIS PROBLEM
VALUE REQUESTED IN CAVE
RUN NO. NS.2 X24C GEOMETRY
X 2 4 C G E O M E T R Y
S1= 0.2000E 00 FT
S2= 0.1000E 00 FT
W1= 0.3000E-01 FT
W3= 0.3000E-01 FT
W5= 0.1500E 00 FT
W7= 0.6000E-01 FT
S2= 0.1800E 00 FT
S4= 0.4000E-01 FT
W2= 0.3000E-01 FT
W4= 0.3000E-01 FT
W6= 0.3000E-01 FT
CONTACT RESISTANCE(1)= 0.5000E-02 BTU/FT**2-SEC-R
CONTACT RESISTANCE(2)= 0.5000E-02 BTU/FT**2-SEC-R
CONTACT RESISTANCE(3)= 0.2000E-01 BTU/FT**2-SEC-R
CONTACT RESISTANCE(4)= 0.2000E-01 BTU/FT**2-SEC-R
EMISSIVITY= 0.8
RADIATION BACKGROUND T= 0.0 DEG.R
I DELX(I)
1 0.3000E-01 2 0.1000E-01 3 0.3000E-01 4 0.1000E-01 5 0.3000E-01
6 0.3000E-01 7 0.1000E-01 8 0.3000E-01 9 0.3000E-01 10 0.1000E-01
11 0.3000E-01 12 0.1000E-01 13 0.3000E-01
I DELY(I)
1 0.1500E-01 2 0.3000E-01 3 0.3000E-01 4 0.3000E-01 5 0.3000E-01
6 0.3000E-01 7 0.3000E-01 8 0.3000E-01 9 0.3000E-01 10 0.3000E-01
11 0.3000E-01 12 0.3000E-01 13 0.1500E-01

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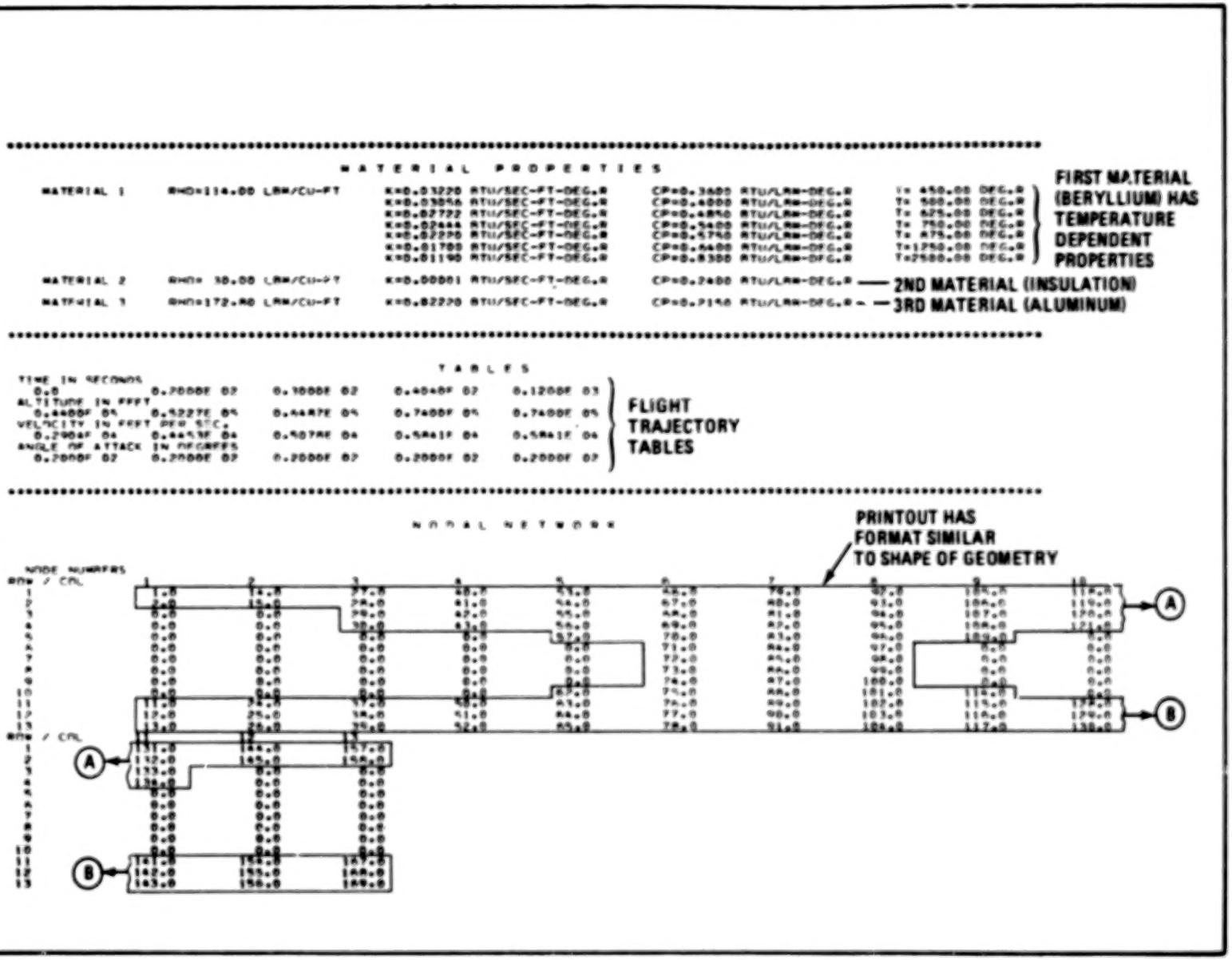
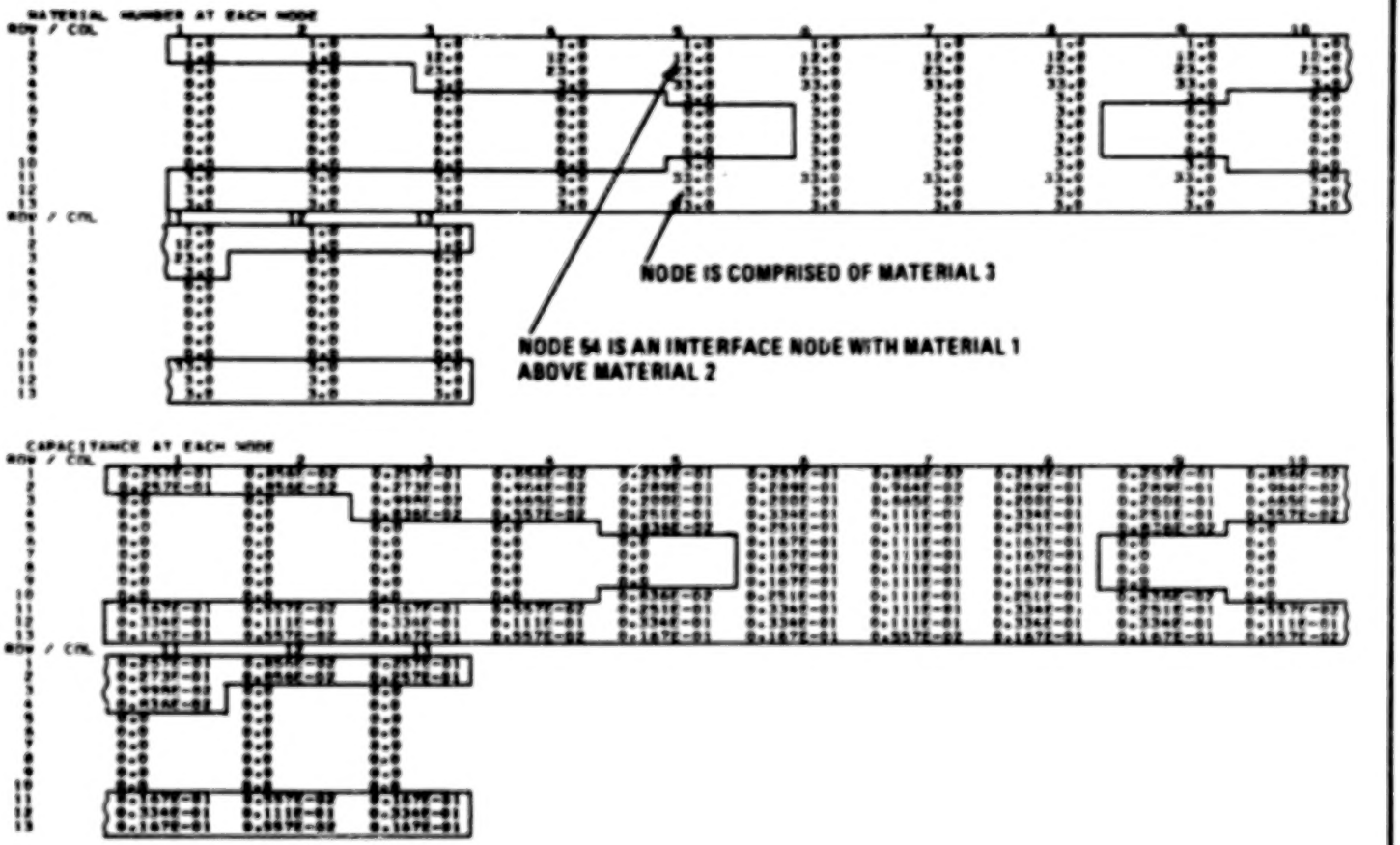
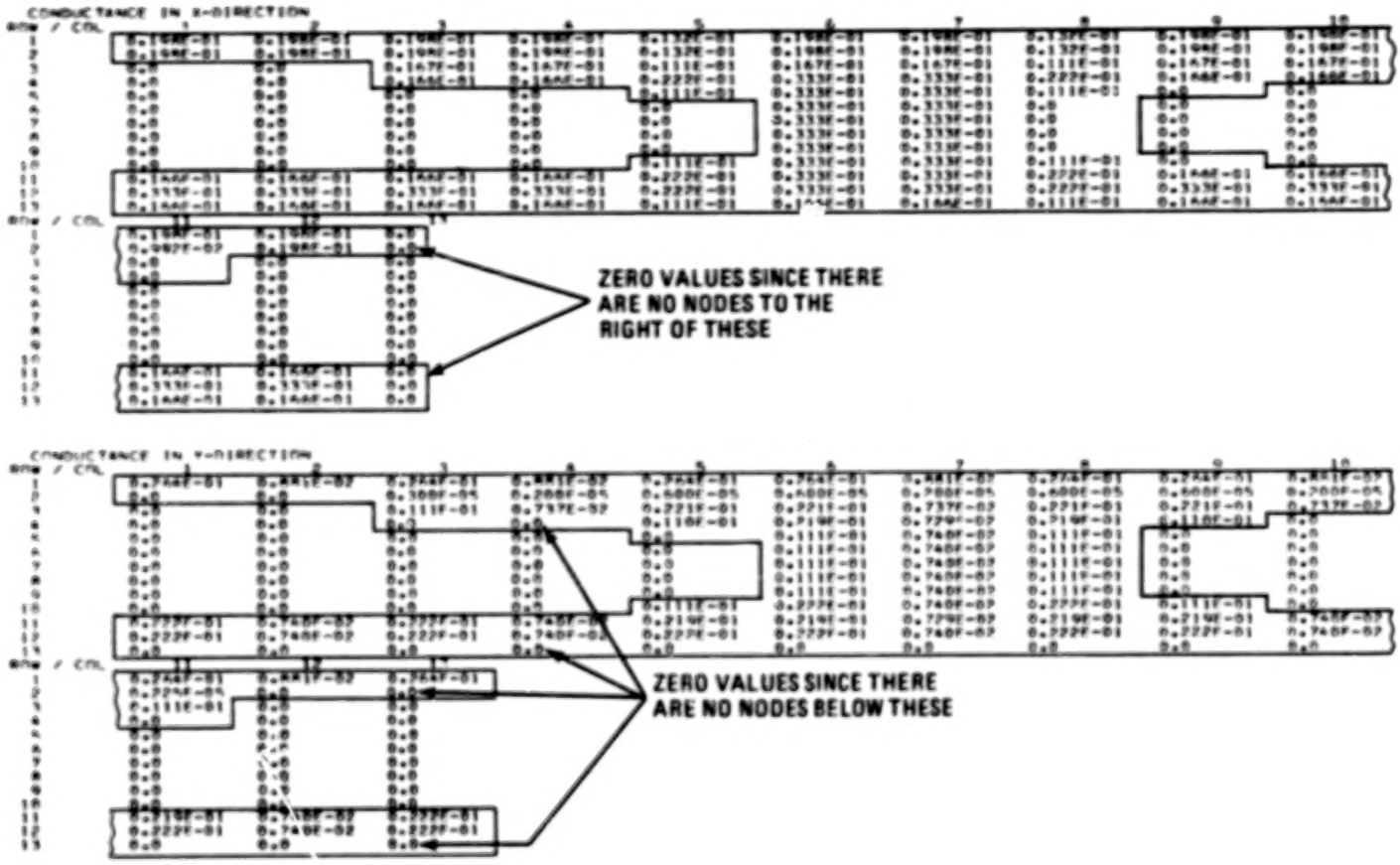
SHEET 4.6 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12)(SHEET 2 OF 9)


MATERIAL PROPERTIES
MATERIAL 1 RHO=114.00 LBM/CU-FT
K=0.53220 BTU/SEC-FT-DEG.R CP=0.3600 BTU/LBM-DEG.R T= 450.00 DEG.R
K=0.53056 BTU/SEC-FT-DEG.R CP=0.4000 BTU/LBM-DEG.R T= 500.00 DEG.R
K=0.52722 BTU/SEC-FT-DEG.R CP=0.4850 BTU/LBM-DEG.R T= 625.00 DEG.R
K=0.52444 BTU/SEC-FT-DEG.R CP=0.5400 BTU/LBM-DEG.R T= 750.00 DEG.R
K=0.52220 BTU/SEC-FT-DEG.R CP=0.5750 BTU/LBM-DEG.R T= 875.00 DEG.R
K=0.51700 BTU/SEC-FT-DEG.R CP=0.6400 BTU/LBM-DEG.R T=1250.00 DEG.R
K=0.51190 BTU/SEC-FT-DEG.R CP=0.8300 BTU/LBM-DEG.R T=2500.00 DEG.R
FIRST MATERIAL (BERYLLIUM) HAS TEMPERATURE DEPENDENT PROPERTIES
MATERIAL 2 RHO= 10.00 LBM/CU-FT K=0.00001 BTU/SEC-FT-DEG.R CP=0.2400 BTU/LBM-DEG.R
2ND MATERIAL (INSULATION)
MATERIAL 3 RHO=172.40 LBM/CU-FT K=0.82220 BTU/SEC-FT-DEG.R CP=0.2150 BTU/LBM-DEG.R
3RD MATERIAL (ALUMINUM)
TABLES
TIME IN SECONDS
0.0 0.20000E 02 0.10000E 02 0.40405E 02 0.12000E 03
ALTITUDE IN FEET
0.44000E 05 0.52270E 05 0.44870E 05 0.74000E 05 0.74000E 05
VELOCITY IN FEET PER SEC.
0.29040E 04 0.44538E 04 0.50780E 04 0.58410E 04 0.58410E 04
ANGLE OF ATTACK IN DEGREES
0.20000E 02 0.20000E 02 0.20000E 02 0.20000E 02 0.20000E 02
FLIGHT TRAJECTORY TABLES
NODAL NETWORK
PRINTOUT HAS FORMAT SIMILAR TO SHAPE OF GEOMETRY
NODE NUMBERS
ROW / COL
A
B

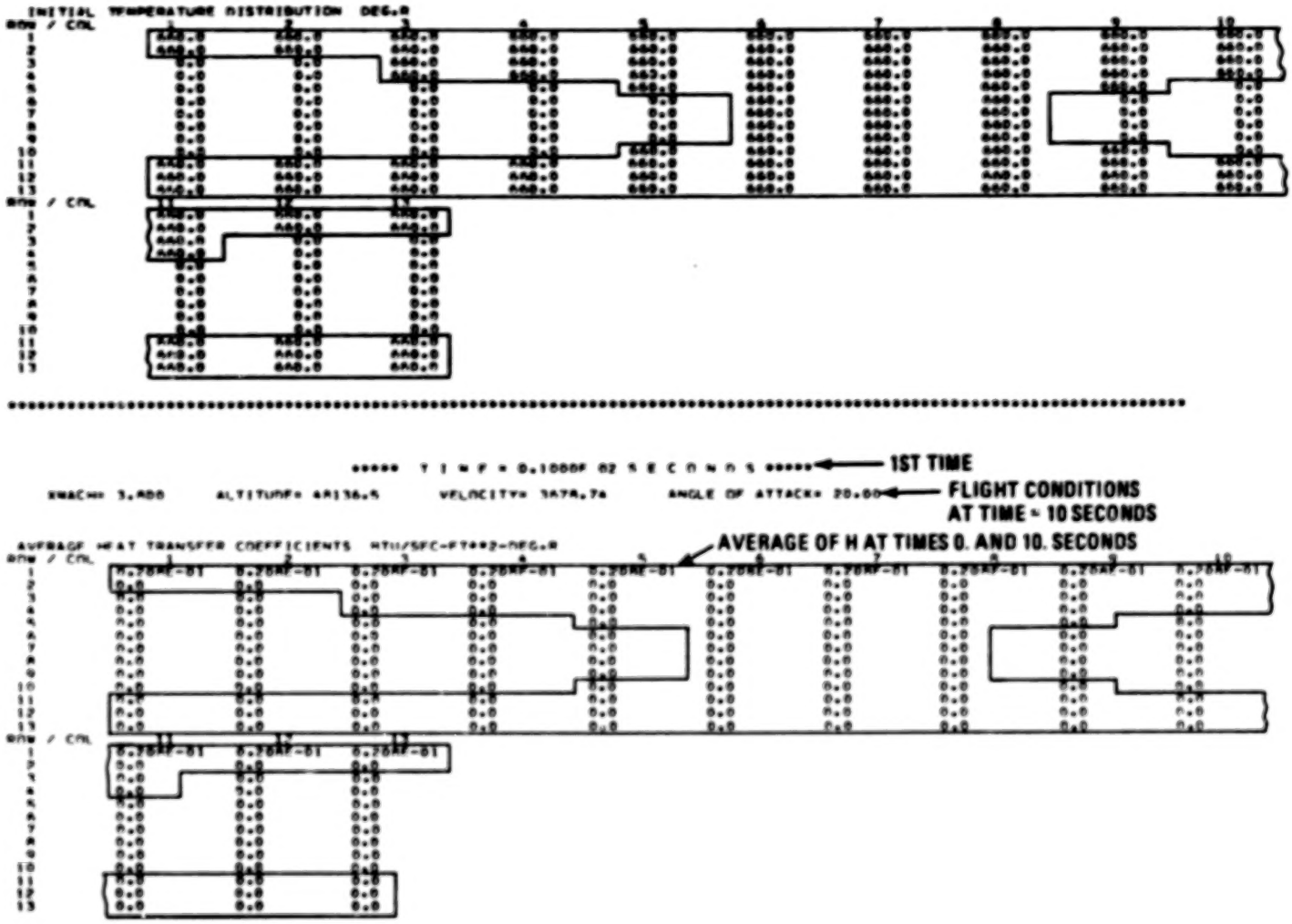
SHEET 4.6 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 3 OF 8)


MATERIAL NUMBER AT EACH NODE
ROW / COL
NODE IS COMPRISED OF MATERIAL 3
NODE 54 IS AN INTERFACE NODE WITH MATERIAL 1
ABOVE MATERIAL 2
CAPACITANCE AT EACH NODE
ROW / COL

SHEET 4.6 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 4 OF 9)


CONDUCTANCE IN X-DIRECTION
ROW / COL
ZERO VALUES SINCE THERE ARE NO NODES TO THE RIGHT OF THESE
CONDUCTANCE IN Y-DIRECTION
ROW / COL
ZERO VALUES SINCE THERE ARE NO NODES BELOW THESE

SHEET 4.6 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 5 OF 9)


INITIAL TEMPERATURE DISTRIBUTION DEG.R
ROW / COL
TIME = 0.10000 02 SECONDS
1ST TIME
XMACH= 3.400 ALTITUDE= 48136.5 VELOCITY= 3478.7a ANGLE OF ATTACK= 20.00
FLIGHT CONDITIONS AT TIME = 10 SECONDS
AVERAGE HEAT TRANSFER COEFFICIENTS HTU/SEC-FT**2-DEG.R
AVERAGE OF H AT TIMES 0. AND 10. SECONDS

SHEET 4.8 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 8 OF 9)

AVERAGE OF HA AT TIME 0 AND 10.

AVERAGE HEAT TRANSFER COUPLINGS BTU/SEC-DEG.R

ROW / COL	1	2	3	4	5	6	7	8	9	10
1	0.623E-03	0.208E-03	0.623E-03	0.208E-03	0.623E-03	0.623E-03	0.208E-03	0.623E-03	0.623E-03	0.208E-03
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ROW / COL	11	12	13
1	0.623E-03	0.208E-03	0.623E-03
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0

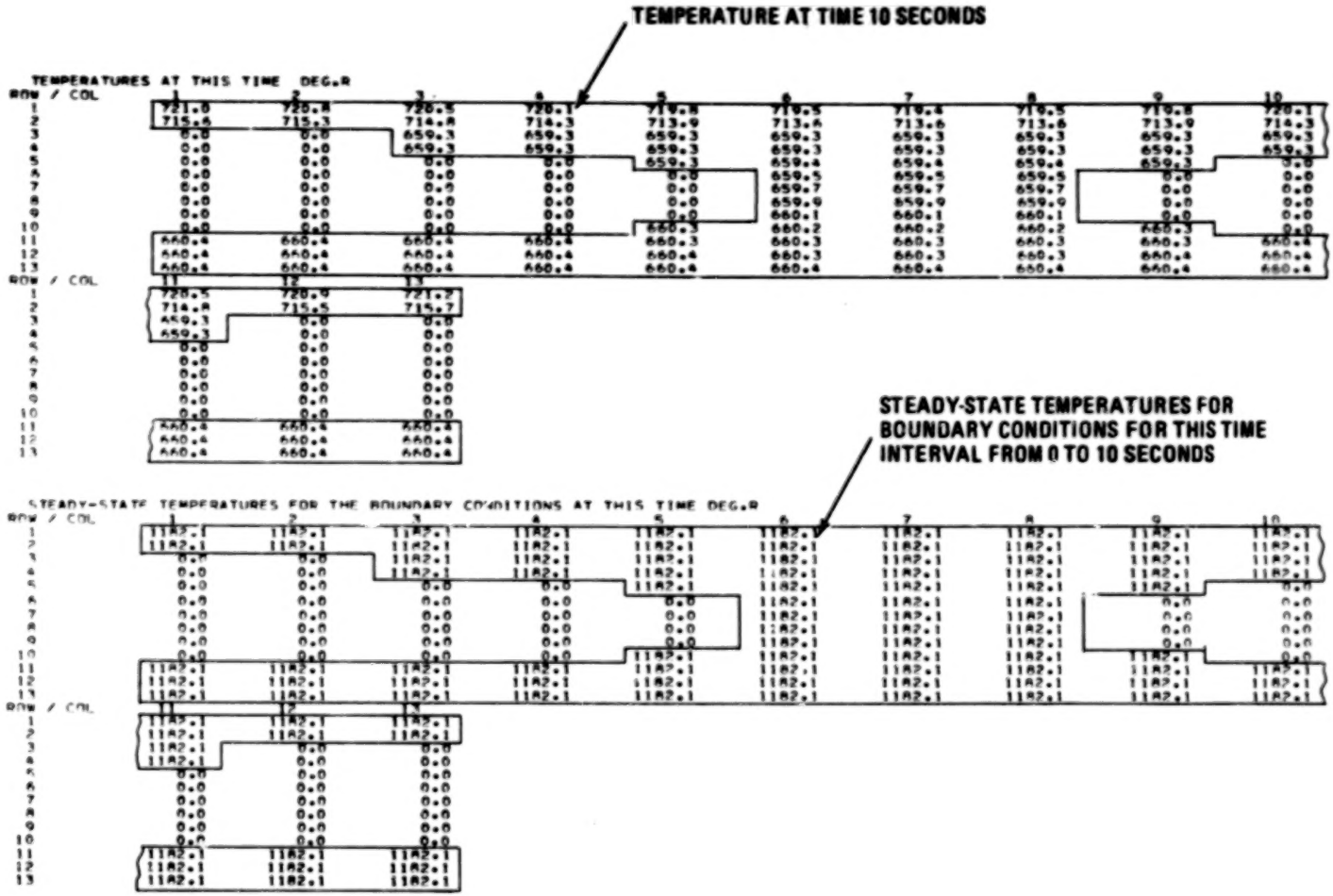
AVERAGE OF T_{AW} AT TIME 0 AND 10.

AVERAGE ADIABATIC WALL TEMPERATURE DEG.R

ROW / COL	1	2	3	4	5	6	7	8	9	10
1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1	11A2.1
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ROW / COL	11	12	13
1	11A2.1	11A2.1	11A2.1
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0

SHEET 4.8 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 7 OF 9)


TEMPERATURE AT TIME 10 SECONDS
TEMPERATURES AT THIS TIME DEG.R
STEADY-STATE TEMPERATURES FOR BOUNDARY CONDITIONS FOR THIS TIME INTERVAL FROM 0 TO 10 SECONDS
STEADY-STATE TEMPERATURES FOR THE BOUNDARY CONDITIONS AT THIS TIME DEG.R

SHEET 4.6 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 8 OF 9)

INTEGRATED HEAT INPUT AT EACH NODE BTU

ROW / COL	1	2	3	4	5	6	7	8	9	10
1	2.9	1.8	2.9	1.8	2.9	2.9	1.8	2.9	2.9	1.8
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ROW / COL	11	12	13
1	2.9	1.8	2.9
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0

NEXT TIME

***** T I M E = 0.2000E 02 S E C O N D S *****

XMACH= 6.400 ALTITUDE= 52273.0 VELOCITY= 4693.24 ANGLE OF ATTACK= 20.00

AVERAGE HEAT TRANSFER COEFFICIENTS BTU/SEC-FT**2-DEG.R

ROW / COL	1	2	3	4	5	6	7	8	9	10
1	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01	0.242E-01
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

ROW / COL	11	12	13
1	0.242E-01	0.242E-01	0.242E-01
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0

SHEET 4.8 OUTPUT DATA FOR X-24C GEOMETRY PROBLEM (REFER TO FIG. 12) (SHEET 9 OF 9)

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Section 5

GENERAL GEOMETRY

5.1 DISCUSSION

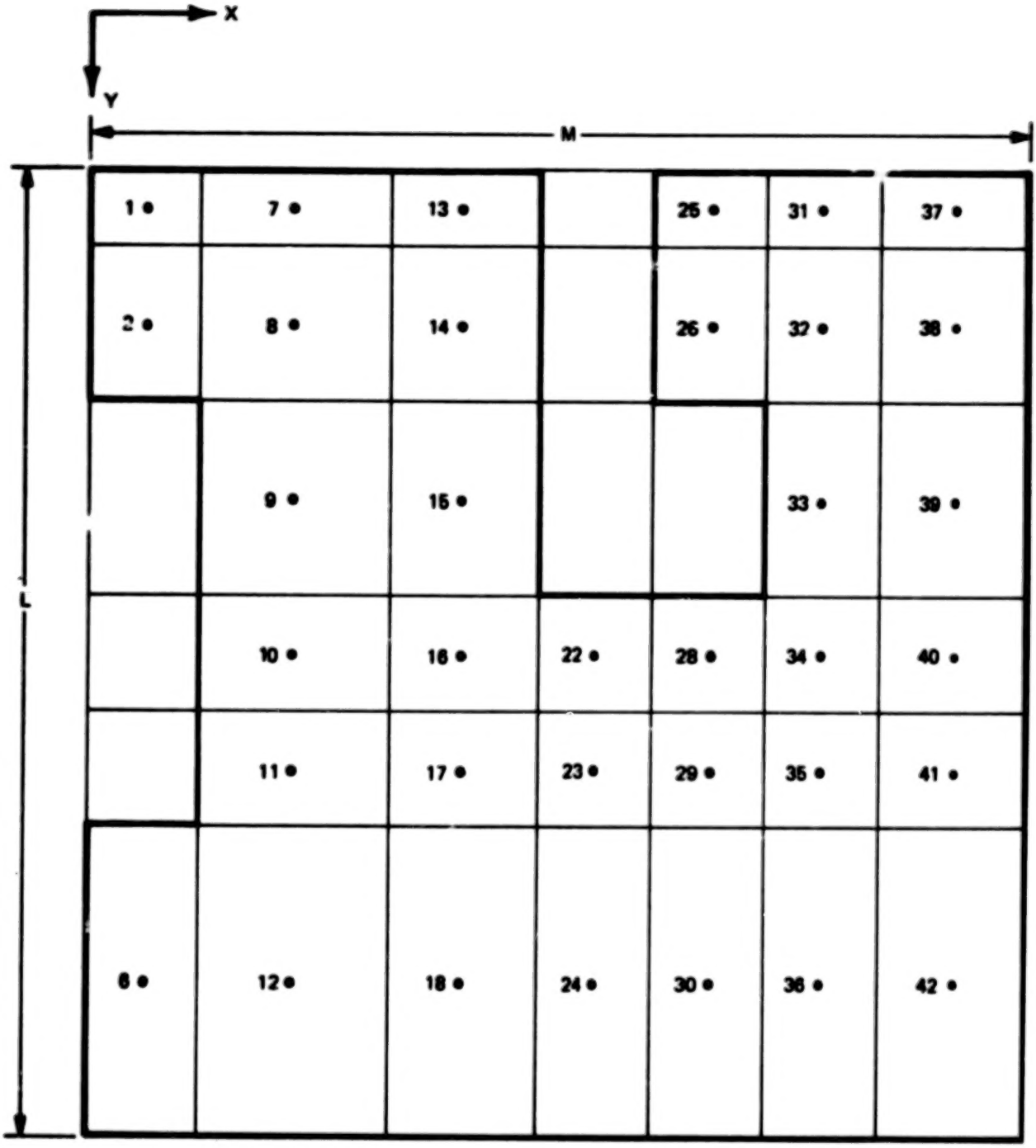
This section describes the general geometry capabilities of CAVE and gives the input data format.

When exercising the general geometry option of CAVE, the user must do the usual tedious and laborious calculations associated with setting up a thermal network. Namely, the volume of each node, the cross-sectional area divided by the X distance between adjacent nodes, and the cross-sectional area divided by the Y distance between adjacent nodes all must be supplied as input data to the code. The code will then multiply the volumes by ρC_p to obtain the node capacitances. The area over distances will be multiplied by the thermal conductivity to obtain the conductances. Multimaterial problems can easily be handled by supplying capacitances instead of volumes and conductances instead of area over distances as input data. The material properties ρ, C_p and k should be input as 1 in this case.

It is possible to simulate within CAVE a convection coupling, either constant or time varying, to each node. The values of the couplings are supplied as input data. Radiation heat transfer is not considered by CAVE when the general geometry option is selected.

The matrix package within CAVE is very efficient from both an execution and storage standpoint. This is made possible by some limitations that have been incorporated into the package. Referring to Figure 13, the limitations may be stated as follows:

1. There exists a conductance coupling between node I and node I + 1, e.g., nodes 16 and 17. (The value of the coupling may be zero.) One exception is when node I is in the bottom row. For example, nodes 12 and 13 are not coupled.



L = 6, M = 7, N = 6 x 7 = 42

FIG. 13 GRID NETWORK FOR A GENERAL GEOMETRY PROBLEM

2. There exists a conductance coupling between node I and node I + L where L is the number of elements in the Y direction, e.g., nodes 16 and 22. (The value of the coupling may be zero.) One exception to this is when node I is in the right column. Since there is no node I + L, there can be no conductance coupling.

3. Node numbering is done sequentially starting with 1 and going to n, the number of nodes. Numbers run columnwise starting at the top of the left column with 1, the node below it 2 and the node to the right of it 1 + L.

4. No other conductance couplings exist. That is, node I will have at most couplings with nodes I-1, I+1, I-L and I+L. It cannot be coupled to any other nodes.

The last assumption precludes, for example, a nodal network having the couplings shown in Figure 14.

On occasion, fictitious nodes are present within the network. They are forced into the network by virtue of the above assumptions regarding the couplings. No input data is required for these nodes since they are not an active part of the problem. Nodes 3, 4, 5, 19, 20, 21 and 27 (given in Fig. 13) are such fictitious nodes.

To visualize the class of problems that CAVE can most readily accommodate, consider a rectangular grid network that gets cut up to form different shapes. If the grid network is drawn on a rubber sheet and that sheet is stretched at will, any resulting figure could be analyzed by CAVE.

The following subsections give the input data format and definition of input variables for running a general problem.

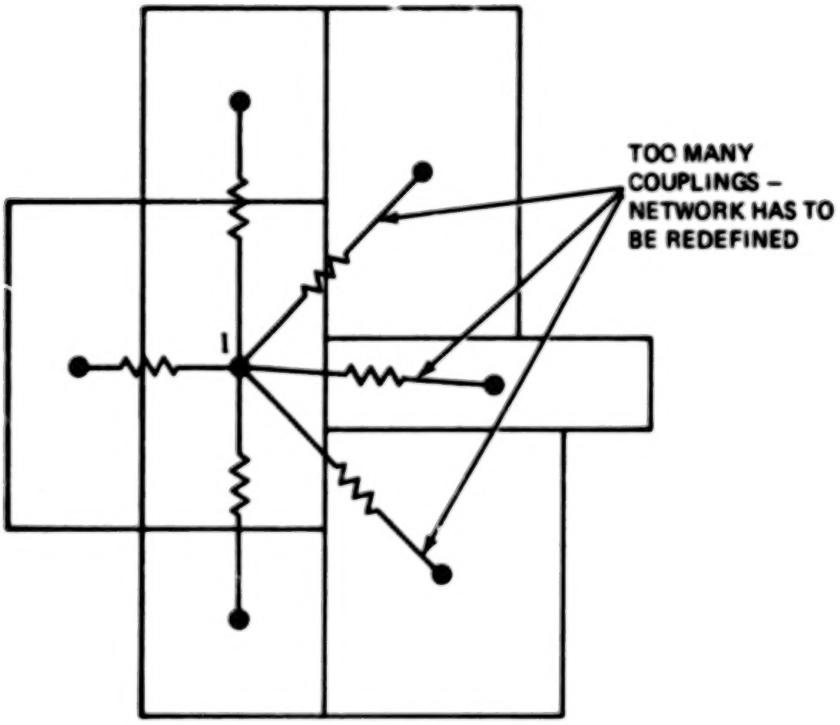


FIG. 14 PORTION OF A GRID NETWORK THAT CANNOT BE HANDLED

5.2 INPUT DATA FORMAT FOR GENERAL GEOMETRY (Fig. 13)

Basic Geometry Card

- JGEO, L, M, NE* (4I5)

  JGEO = -1 (selects general geometry option)

  L = maximum number of elements in the Y direction

  M = maximum number of elements in the X direction

  NE = number of dominant eigenvalues to be used in solution (e.g., a typical number is 3)

Title Card

- Run identification, comments, etc. (5A10)
- Blank Card

Material Properties Cards

- MAT (I5)
- NMAT1, RHO1, CONAV1, CPAV1 (I10, 3F10.5)
- TMAT1(1), TMAT1(2), ..., TMAT1(NMAT1) (8E10.0) } omit if NMAT1=0
- CONDT1(1), CONDT1(2), ..., CONDT1(NMAT1) (8E10.0) } omit if NMAT1=0
- CPT1(1), CPT1(2), ..., CPT1(NMAT1) (8E10.0) } omit if NMAT1=0

(If MAT = 2 include the cards:)

- NMAT2, RHO2, CONAV2, CPAV2 (I10.0,3F10.5)
- TMAT2(1), TMAT2(2), ..., TMAT2(NMAT2) (8E10.0) } omit if NMAT=2
- CONDT2(1), CONDT2(2), ..., CONDT2(NMAT2) (8E10.0) } omit if NMAT=2
- CPT2(1), CPT2(2), ..., CPT2(NMAT2) (8E10.0) } omit if NMAT=2

MAT = number of materials (1, 2 or 3)

NMAT1 = number of entries in properties table (maximum of 10)
NMAT1 = 0 for constant properties

RHO1 = density of material 1, lbm/cu-ft

CONAV1 = average thermal conductivity of material 1 (used when NMAT1 = 0), Btu/ft-sec-°R

CPAV1 = average specific heat of material 1 (used when NMAT1 = 0), Btu/lbm-°R

*Current dimension limitations require that the product LXM not exceed 200.

TMAT1(I) = temperatures in thermal properties table for which CONDT1(I) and CPT1(I) are given; I = 1, 2, ..., NMAT1, °R

CONDT1(I) = thermal conductivity of material 1 at temperature TMAT1(I), Btu/ft-sec-°R

CPT1(I) = specific heat of material 1 at temperature TMAT1(I), Btu/lbm-°R

NMAT2, RHO2, CONAV2, etc., same as NMAT1, RHO1, CONAV1, etc., except applied to material 2

Volume Cards

- KODE, I, V(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

V(I) = node volume, cu ft (or optionally ρVC_p), Btu/°R

II = limit for multiple parameter input

JJ = increment for multiple parameter input

(The node number is incremented by the spacing JJ until the limit II is reached. Each node so specified is assigned the same te perature.)

Material Selection Cards

- KODE, I, MATNUM(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

MATNUM(I) = 1 to use material 1 properties

= 2 to use material 2 properties

= 3 when node is at interface between materials 1 and 2 (interface is understood to be parallel to X-axis)

II = limit for multiple parameter input

JJ = increment for multiple paremeter input

Area Over X Cards

- KODE, I, AOVERX(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

AOVERX(I) = cross-sectional area divided by X distance between node I and node I + L, required only for nonzero conductances, ft^2/ft (or optionally, the conductance between nodes I and I + L), Btu/sec-°R

II = limit for multiple parameter input

JJ = increment for multiple parameter input

Area Over Y Cards

- KODE, I, AOVERY(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

AOVERX(I) = cross-sectional area divided by Y distance between node I and node I + 1, required only for nonzero conductances, ft^2/ft (or optionally, the conductance between nodes I and I + 1), Btu/sec-°R

II = limit for multiple parameter input

JJ = increment for multiple parameter input

Convection Coupling Cards (2I5,E10.0,2I5)

- KODE, I, HA(I), II, JJ
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

HA(I) = convective coupling between node I and TAW(I), Btu/sec-°R
(or optionally, a negative number to indicate that the coupling is time varying. The absolute value of the negative number is then the number of the dependent variable in the convection table that gives HA(I) as a function of time)

II = limit for multiple parameter input

JJ = increment for multiple parameter input

Adiabatic Wall Temperature Cards

- KODE, I, TAW(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

TAW(I) = adiabatic wall temperature associated with node I, °R
(or optionally, a negative number to indicate that the adiabatic wall temperature is time varying. The absolute value of the negative number is then the number of the dependent variable in the convection table that gives TAW as a function of time)

II = limit for multiple parameter input

JJ = increment for multiple parameter input

Convection Table Cards

- KODE, L1, L2, TITLE (3I5,A65)
- TIME(1), TIME(2), ..., TIME(L1) (7E10.0)
- D1(1), D1(2), ..., D1(L1) (7E10.0)
- D2(1), D2(2), ..., D2(L1) (7E10.0)
- . . .
- . . .
- . . .
- DL2(1), DL2(2), ..., DL2(L1) (7E10.0)
- 11100 (I5)

KODE = 0 or blank

L1 = number of values of time, the independent variable (L1 ≤ 250/L2)

L2 = number of dependent variables (1 ≤ L2 ≤ 20)

TIME(I) = time in convection table I = 1, 2, ... L1, sec

D1(I) = first dependent variable to represent either HA or TAW associated with a node or nodes at TIME(I)

- . . .
- . . .
- . . .

DL2(I) = last dependent variable to represent either HA or TAW associated with a node or nodes at TIME(I)

Initial Temperature Cards

- KODE, I, T(I), II, JJ (2I5,E10.0,2I5)
- . . .
- . . .
- . . .
- 11100 (I5)

KODE = 0 or blank

I = node number

T(I) = node initial temperature, °R

II and JJ = the node number is incremented by the spacing JJ until the limit II is reached. Each node so specified is assigned the same temperature

Time Interval Cards

- NTIMES (I10)
- TIMES(1), TIMES(2), ... TIMES(NTIMES) (8E10.0)

NTIMES = number of points in time intervals array ($2 \leq NTIMES \leq 50$)

TIMES(1) = initial time (usually equals 0.), sec

TIMES(I) = time at which temperatures will be calculated and printed out I = 2, 3, ..., NTIMES, sec

5.3 SAMPLE PROBLEM FOR GENERAL GEOMETRY (Fig. 15)

The general capabilities of CAVE are illustrated via the simple grid network shown in Figure 15. While CAVE can handle much more arbitarily shaped geometries than this one, all the basic ingredients are present here.

As Figure 15 shows, two materials were used: a layer of beryllium across the top that was being aerodynamically heated and the remainder aluminum with a portion of its boundary being cooled. The convective coefficients and adiabatic wall temperatures associated with the aerodynamic heating are shown as functions of distance and time in Figure 15. The values of $h\Delta X$ and T_{AW} for the different times at nodes 1, 9, 17, 25 and 33 were provided as input data to represent the aerodynamic heating.

The following pages present listings of the input data used for this problem and the resulting output data. The input data for this general type of geometry is seen to be more extensive than for one of the built-in configurations. This is due to the need to input for each node a volume, material number, condition area over distances, convection coupling and adiabatic wall temperature, in addition to the usual material properties initial temperatures and time step intervals.

The output from CAVE for a general geometry problem is very similar to that of a built-in geometry. At the end of every time interval there is a printout of the convection couplings, adiabatic wall temperatures, steady-state temperatures and integrated heat inputs. No printout of convective coefficients appears since the general geometry option deals strictly with the coupling, $h\Delta x$ and not with h alone. (The input and output have been annotated to assist the reader.)

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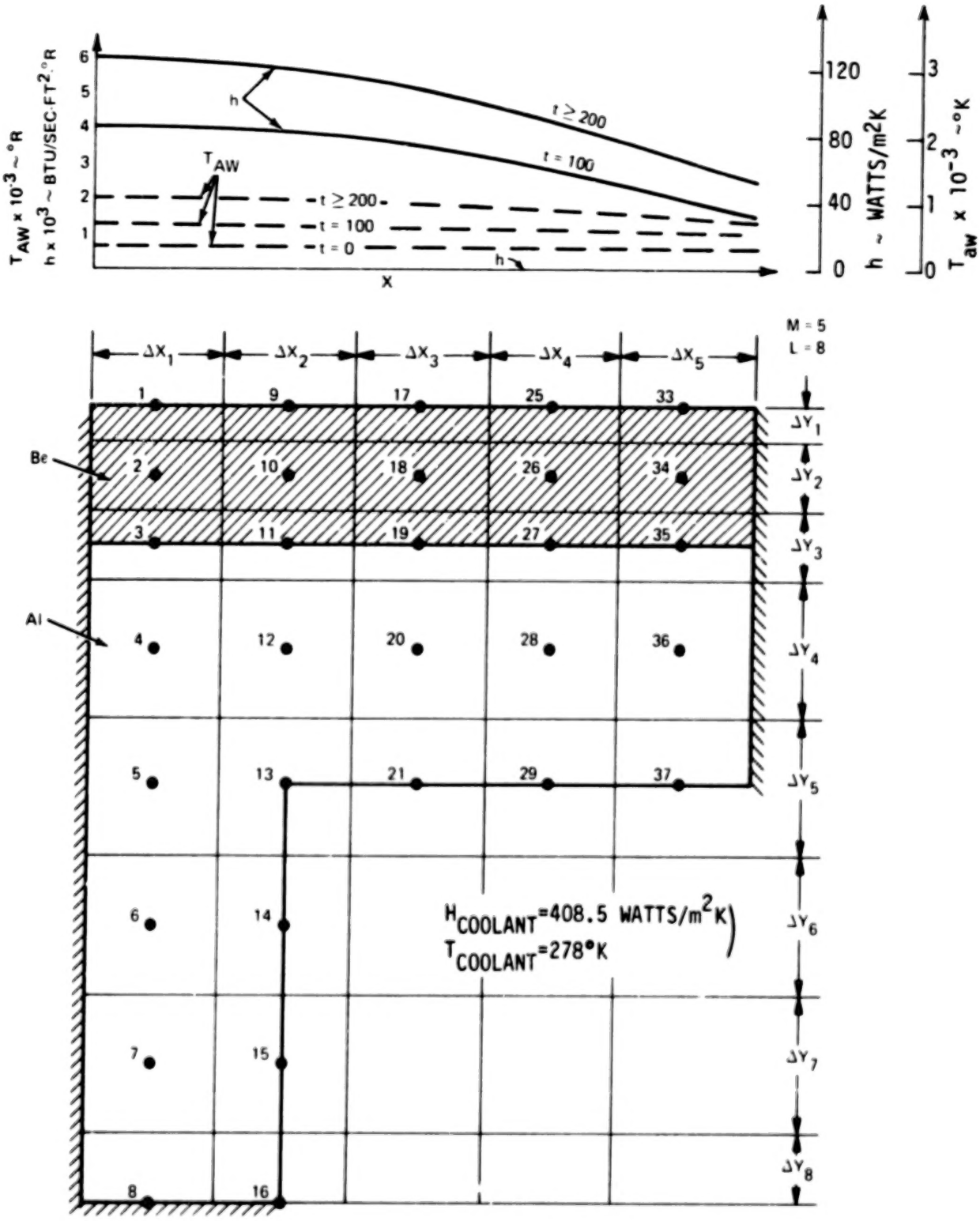
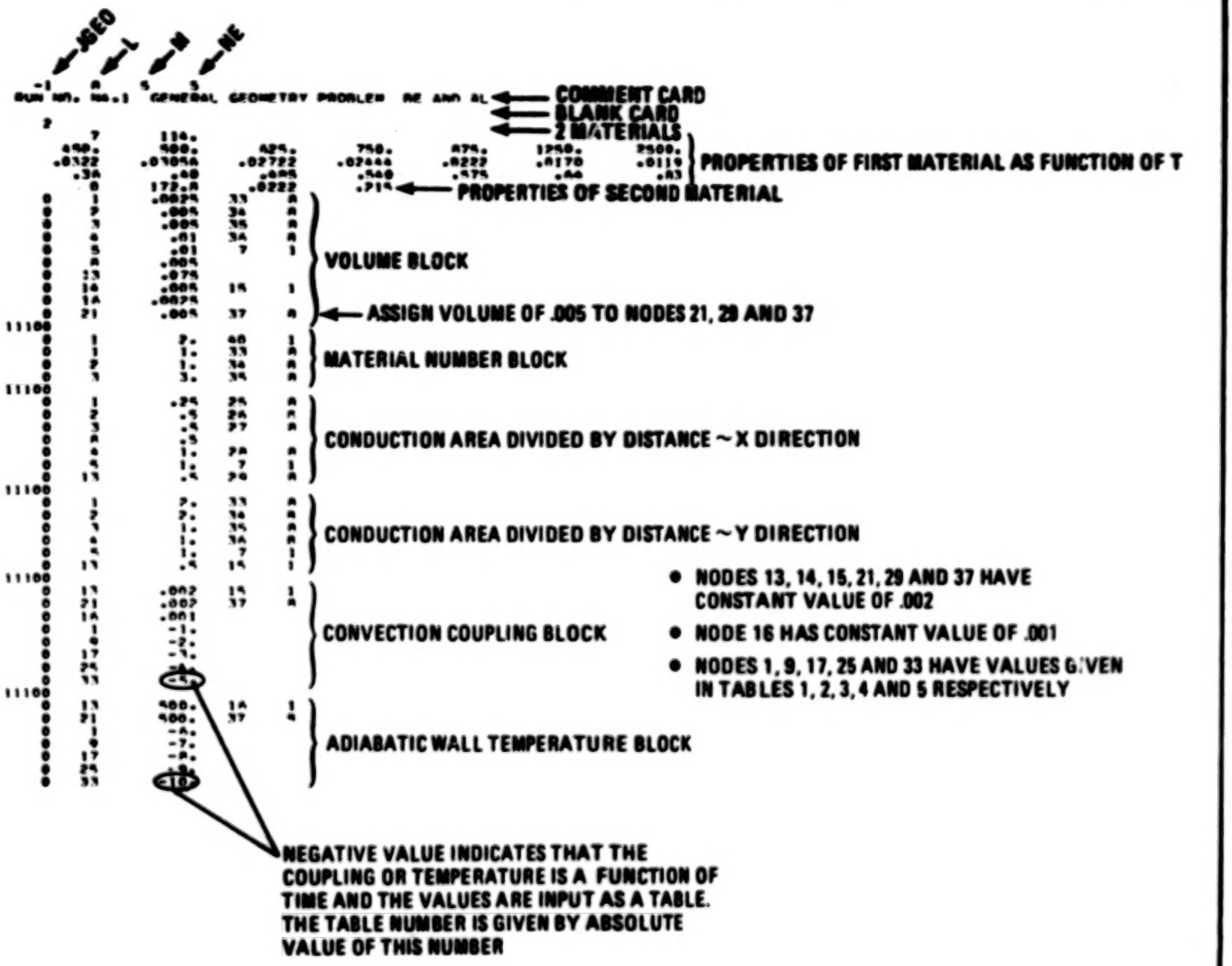
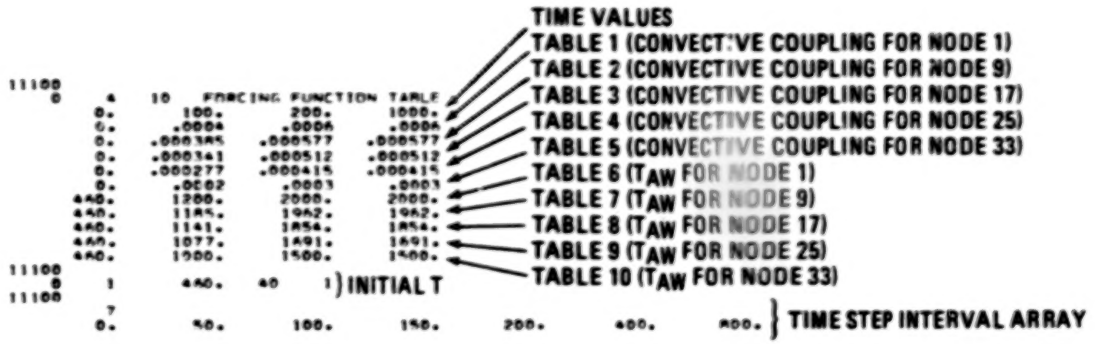


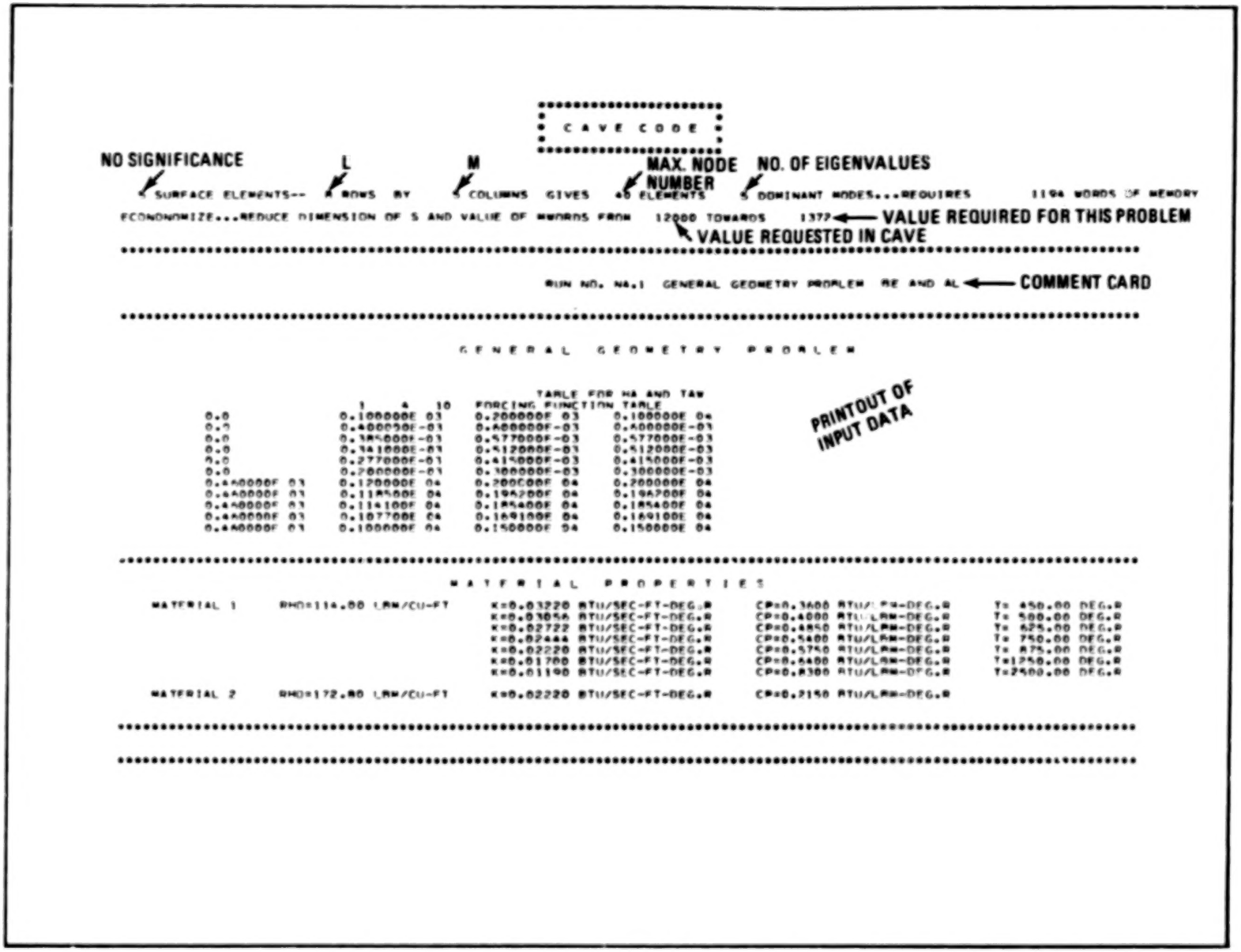
FIG. 15 GENERAL GEOMETRY PROBLEM


JSEQ
L
M
NE
COMMENT CARD
BLANK CARD
2 MATERIALS
PROPERTIES OF FIRST MATERIAL AS FUNCTION OF T
PROPERTIES OF SECOND MATERIAL
VOLUME BLOCK
ASSIGN VOLUME OF .005 TO NODES 21, 29 AND 37
MATERIAL NUMBER BLOCK
CONDUCTION AREA DIVIDED BY DISTANCE ~ X DIRECTION
CONDUCTION AREA DIVIDED BY DISTANCE ~ Y DIRECTION
CONVECTION COUPLING BLOCK
• NODES 13, 14, 15, 21, 29 AND 37 HAVE CONSTANT VALUE OF .002
• NODE 16 HAS CONSTANT VALUE OF .001
• NODES 1, 9, 17, 25 AND 33 HAVE VALUES GIVEN IN TABLES 1, 2, 3, 4 AND 5 RESPECTIVELY
ADIABATIC WALL TEMPERATURE BLOCK
NEGATIVE VALUE INDICATES THAT THE COUPLING OR TEMPERATURE IS A FUNCTION OF TIME AND THE VALUES ARE INPUT AS A TABLE. THE TABLE NUMBER IS GIVEN BY ABSOLUTE VALUE OF THIS NUMBER

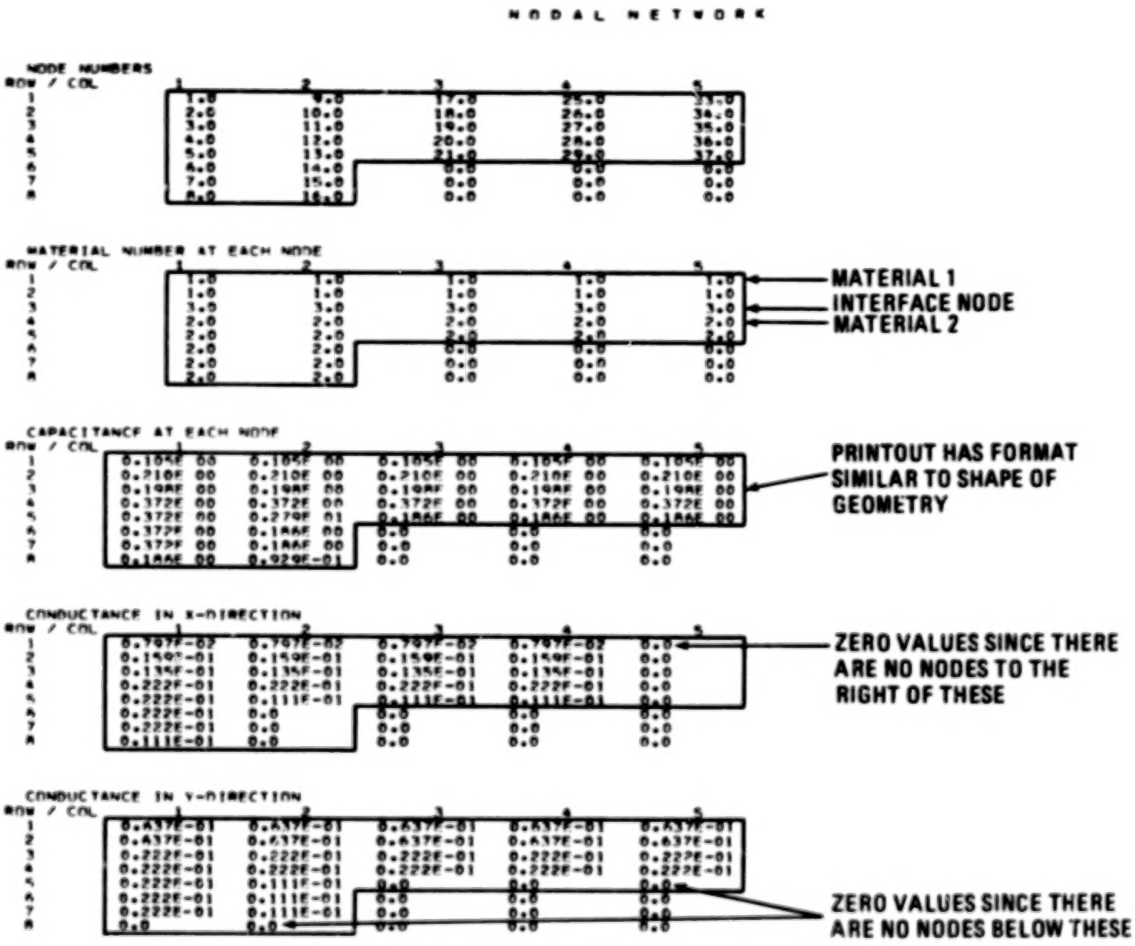
SHEET 5.3 INPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15) (SHEET 1 OF 6)


11100
0 4 10 FORCING FUNCTION TABLE
0. 100. 200. 1000.
0. .0004 .0006 .0006
0. .000385 .000577 .000577
0. .000341 .000512 .000512
0. .000277 .000415 .000415
0. .0002 .0003 .0003
440. 1200. 2000. 2000.
440. 1185. 1962. 1962.
440. 1141. 1854. 1854.
440. 1077. 1691. 1691.
440. 1000. 1500. 1500.
11100
0 1 440. 40 1
11100
7
0. 50. 100. 150. 200. 400. 800.
TIME VALUES
TABLE 1 (CONVECTIVE COUPLING FOR NODE 1)
TABLE 2 (CONVECTIVE COUPLING FOR NODE 9)
TABLE 3 (CONVECTIVE COUPLING FOR NODE 17)
TABLE 4 (CONVECTIVE COUPLING FOR NODE 25)
TABLE 5 (CONVECTIVE COUPLING FOR NODE 33)
TABLE 6 (T_{AW} FOR NODE 1)
TABLE 7 (T_{AW} FOR NODE 9)
TABLE 8 (T_{AW} FOR NODE 17)
TABLE 9 (T_{AW} FOR NODE 25)
TABLE 10 (T_{AW} FOR NODE 33)
INITIAL T
TIME STEP INTERVAL ARRAY

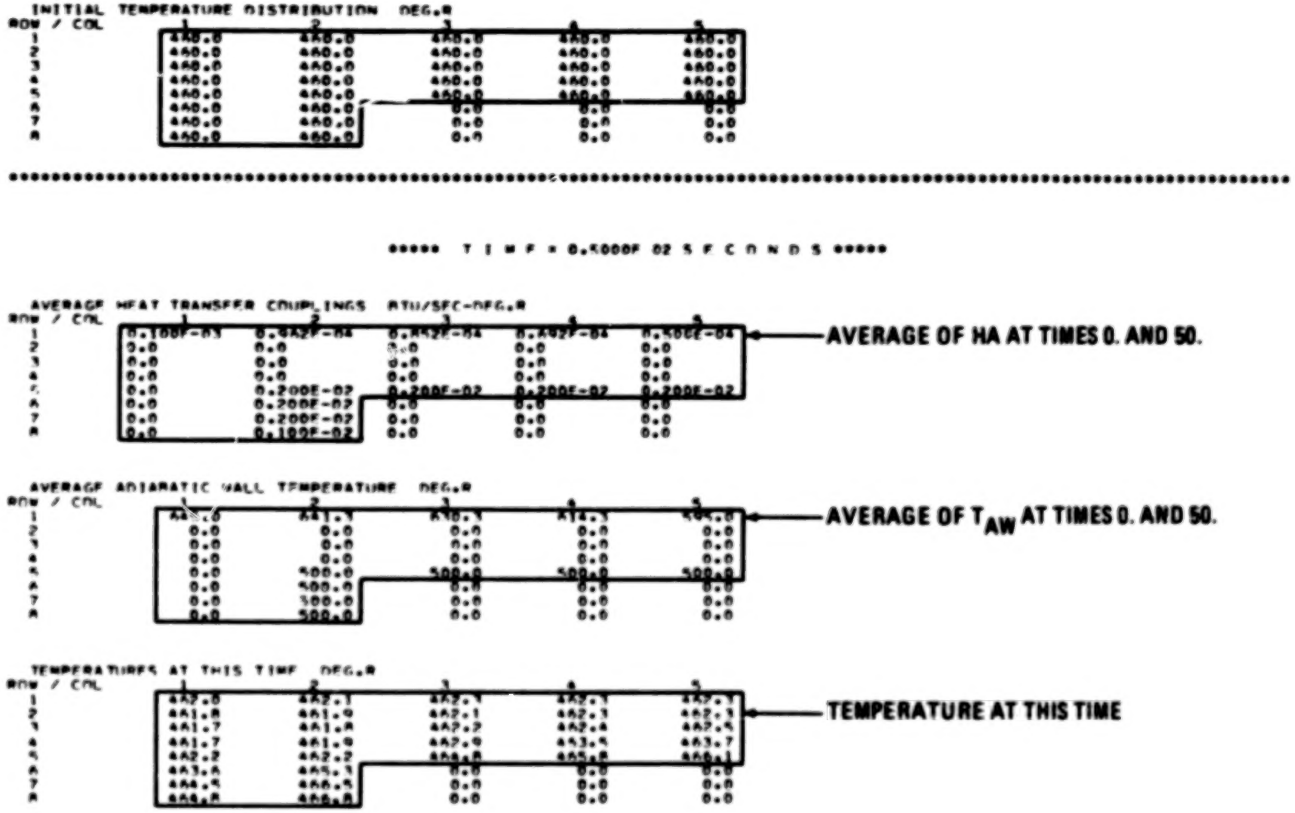
SHEET 5.3 INPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15) (SHEET 2 OF 6)


CAVE CODE
NO SIGNIFICANCE
L
M
MAX. NODE NUMBER
NO. OF EIGENVALUES
4 SURFACE ELEMENTS-- 4 ROWS BY 5 COLUMNS GIVES 40 ELEMENTS 5 DOMINANT MODES...REQUIRES 1194 WORDS OF MEMORY
ECONOMMIZE...REDUCE DIMENSION OF 5 AND VALUE OF MWORDS FROM 12000 TOWARDS 1377
VALUE REQUIRED FOR THIS PROBLEM
VALUE REQUESTED IN CAVE
RUN NO. N4.1 GENERAL GEOMETRY PROBLEM RE AND AL
COMMENT CARD
GENERAL GEOMETRY PROBLEM
TABLE FOR HA AND TAW
1 4 10
FORCING FUNCTION TABLE
0.0 0.1000000E 03 0.2000000E 03 0.1000000E 04
0.0 0.4000000E-01 0.4000000E-01 0.4000000E-01
0.0 0.3850000E-01 0.5770000E-01 0.5770000E-01
0.0 0.3410000E-01 0.5120000E-01 0.5120000E-01
0.0 0.2770000E-01 0.4150000E-01 0.4150000E-01
0.0 0.2000000E-01 0.3000000E-01 0.3000000E-01
0.4400000E 01 0.1200000E 04 0.2000000E 04 0.2000000E 04
0.4400000E 01 0.1185500E 04 0.1982000E 04 0.1982000E 04
0.4400000E 01 0.1141000E 04 0.1854000E 04 0.1854000E 04
0.4400000E 01 0.1077000E 04 0.1691000E 04 0.1691000E 04
0.4400000E 01 0.1000000E 04 0.1500000E 04 0.1500000E 04
PRINTOUT OF INPUT DATA
MATERIAL PROPERTIES
MATERIAL 1 RHO=114.00 LBM/CU-FT
K=0.03220 BTU/SEC-FT-DEG.R CP=0.3600 BTU/LBM-DEG.R T= 450.00 DEG.R
K=0.03056 BTU/SEC-FT-DEG.R CP=0.4000 BTU/LBM-DEG.R T= 500.00 DEG.R
K=0.02722 BTU/SEC-FT-DEG.R CP=0.4850 BTU/LBM-DEG.R T= 625.00 DEG.R
K=0.02444 BTU/SEC-FT-DEG.R CP=0.5400 BTU/LBM-DEG.R T= 750.00 DEG.R
K=0.02220 BTU/SEC-FT-DEG.R CP=0.5750 BTU/LBM-DEG.R T= 875.00 DEG.R
K=0.01700 BTU/SEC-FT-DEG.R CP=0.6400 BTU/LBM-DEG.R T=1250.00 DEG.R
K=0.01190 BTU/SEC-FT-DEG.R CP=0.8300 BTU/LBM-DEG.R T=2500.00 DEG.R
MATERIAL 2 RHO=172.80 LBM/CU-FT
K=0.02220 BTU/SEC-FT-DEG.R CP=0.2150 BTU/LBM-DEG.R

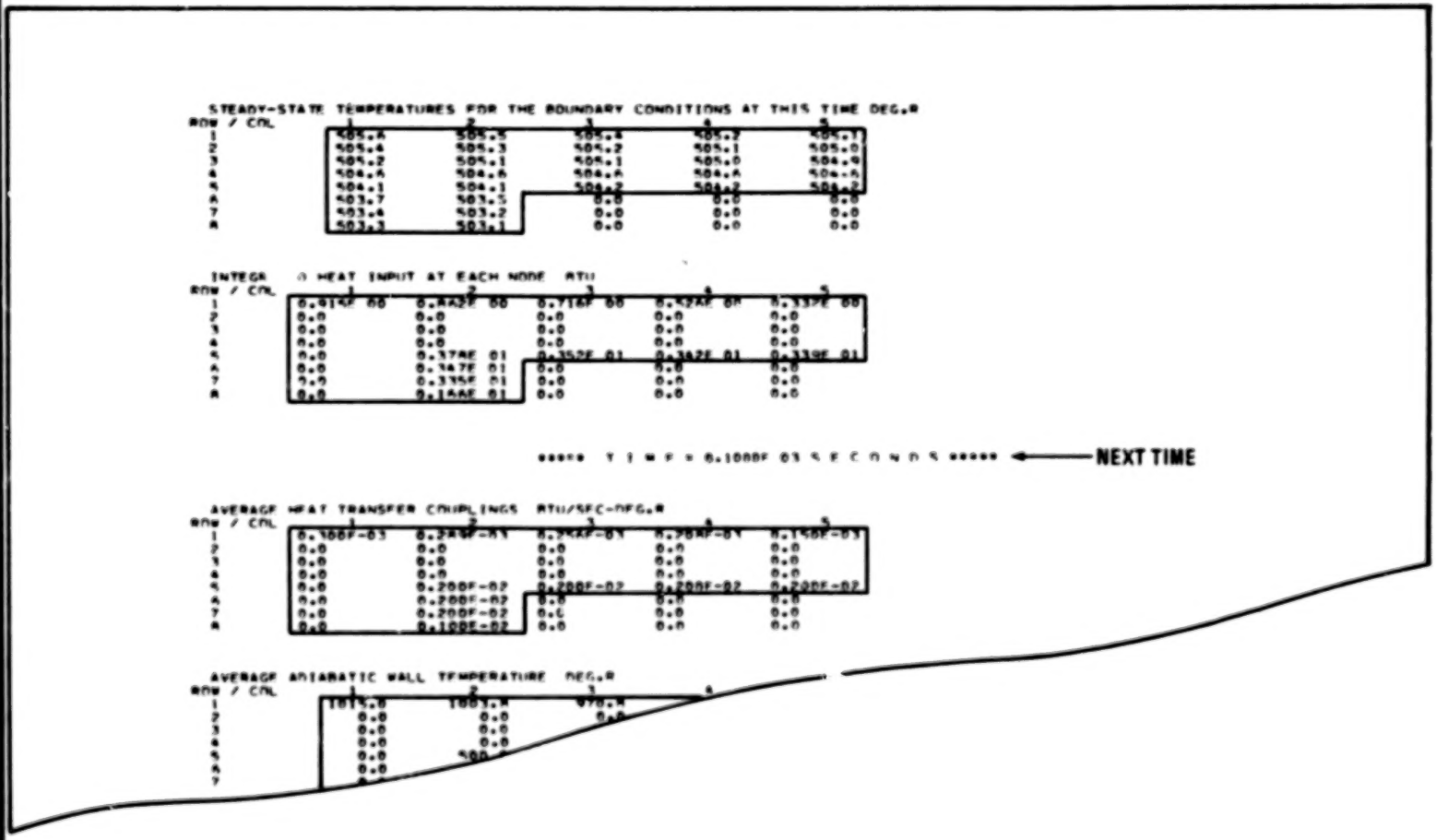
SHEET 5.3 OUTPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15) (SHEET 3 OF 6)


NODAL NETWORK
NODE NUMBERS
ROW / COL
MATERIAL NUMBER AT EACH NODE
MATERIAL 1
INTERFACE NODE
MATERIAL 2
CAPACITANCE AT EACH NODE
PRINTOUT HAS FORMAT SIMILAR TO SHAPE OF GEOMETRY
CONDUCTANCE IN X-DIRECTION
ZERO VALUES SINCE THERE ARE NO NODES TO THE RIGHT OF THESE
CONDUCTANCE IN Y-DIRECTION
ZERO VALUES SINCE THERE ARE NO NODES BELOW THESE

SHEET 5.3 OUTPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15) (SHEET 4 OF 6)



SHEET 5.3 OUTPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15) (SHEET 5 OF 6)


STEADY-STATE TEMPERATURES FOR THE BOUNDARY CONDITIONS AT THIS TIME DEG.R
INTEGR ... HEAT INPUT AT EACH NODE BTU
***** T I M E = 0.1000E 03 S E C O N D S *****
NEXT TIME
AVERAGE HEAT TRANSFER COUPLINGS BTU/SEC-DEG.R
AVERAGE ADIABATIC WALL TEMPERATURE DEG.R

SHEET 5.3 OUTPUT DATA FOR GENERAL GEOMETRY PROBLEM (REFER TO FIG. 15)(SHEET 6 OF 6)

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APPENDIX A

DESCRIPTION OF THE HYBRID ANALYTICAL-NUMERICAL TECHNIQUE

This appendix presents a brief summary of the Hybrid Analytical-Numerical (HAN) technique using, as an example, a one-dimensional, finite thickness, conduction problem. This application provides two services. First it provides a clear exposition of the HAN technique with its attendant matrix operations; and second, it provides insight into the accuracy of the technique since an exact solution to the problem is known. The effect of retaining only the dominant eigenvalues and eigenvectors (E & E) on the accuracy of the solution can be clearly assessed. In NASA CR-2435, Maise and Rossi thoroughly investigated the effect that the number of E & E's have on the accuracy of the h predicted for the inverse problem. In addition, for a direct problem at a particular time, they show the typical errors incurred by neglecting subdominant eigenvalues in a very simple model problem. This appendix explores how the incurred errors vary as a function of time. The solution to the sample problem was obtained using a specially prepared computer code to perform all of the matrix operations and companion calculations. The solution provided a completely independent check on the eigenvalues, eigenvectors and temperatures generated by CAVE.

The problem considered was that of a slab heated by convection on one face and perfectly insulated on the other face. Figure A-1 shows the 10-node network selected to solve the problem; also shown are the properties, temperatures and dimensions used.

In the HAN method, spatial derivatives are replaced by their appropriate finite difference representations and the temporal derivatives are retained as ordinary derivatives. In effect, the problem is subdivided into a number of uniform temperature systems or nodes that are coupled and changing in temperature. Utilizing the notation of Appendix F, the set of ten, first-order, linear, ordinary differential equations for the temperatures at the ten nodes can be written as:

$$M \dot{\underline{T}} = B \underline{T} + \underline{F} \qquad \text{Eq. (A-1a)}$$

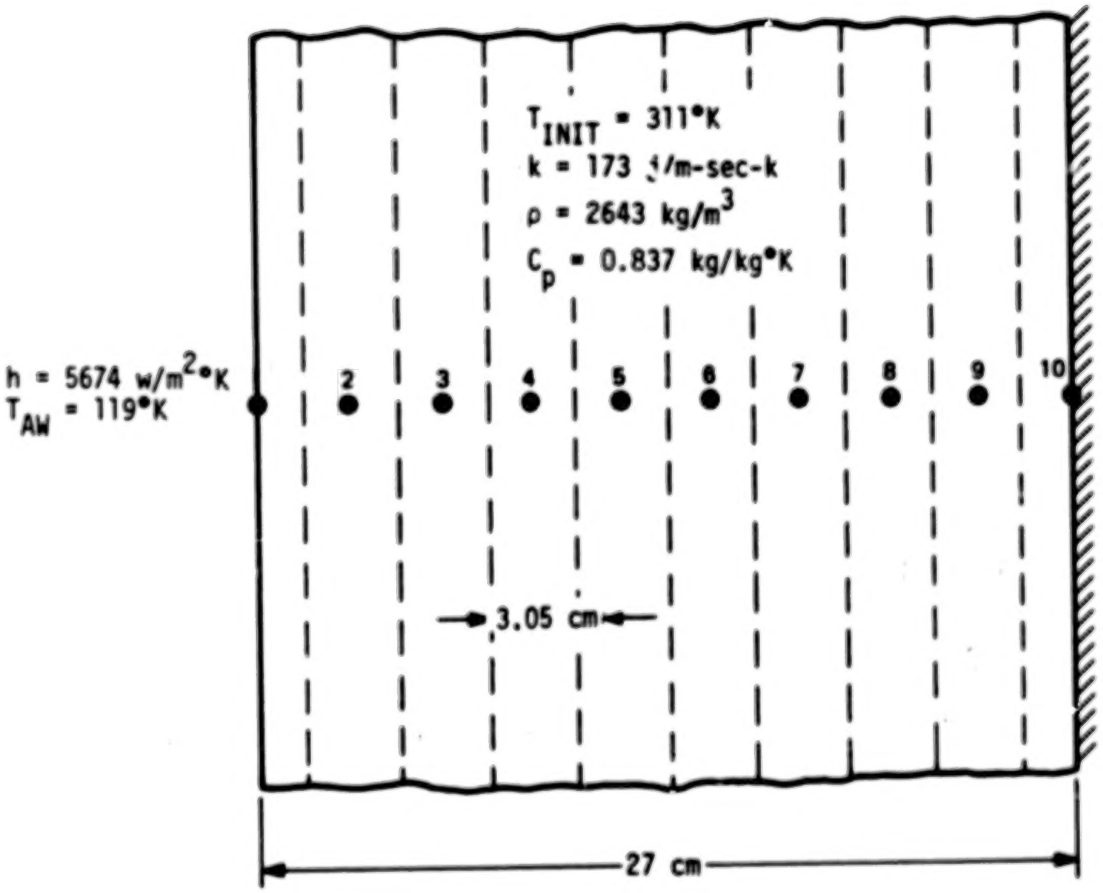


FIG. A-1 NODAL NETWORK FOR ONE-DIMENSIONAL CONDUCTION PROBLEM

subject to the initial condition,

$$\underline{T}^{(0)} = \underline{T}_{init} = \begin{Bmatrix} 100 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 100 \end{Bmatrix} \qquad \text{Eq. (A-1b)}$$

The matrix is a 10 x 10 diagonal matrix associated with the heat capacity of the nodes and is given by:

$$M = \begin{bmatrix} 1.65 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 3.30 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 3.30 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.30 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 3.30 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.30 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.30 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.30 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.30 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.65 \end{bmatrix}$$

where,

$$M_{11} = M_{10\ 10} = \frac{\Delta x}{2} \rho C$$

$$M_{ii} = \Delta x \rho C \text{ for } i = 2, 3, \ldots 9$$

The matrix B, also 10 x 10 in size, is associated with the heat transfer couplings between nodes and is given by:

$$B = \begin{bmatrix} -2000. & 1000. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 1000. & -2000. & 1000. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1000. & -2000. & 1000. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 1000. & -2000. & 1000. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1000. & -2000. & 1000. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1000. & -2000. & 1000. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 1000. & -2000. & 1000. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1000. & -2000. & 1000. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1000. & -2000. & 1000. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1000. & -2000. \end{bmatrix}$$

where,

$$b_{11} = -h - \frac{k}{\Delta x}$$

$$i = 2, 3, \ldots 9$$

$$b_{ii} = -2\frac{k}{\Delta x}$$

$$b_{10\ 10} = \frac{-k}{\Delta x}$$

and,

$$b_{i,i+1} = b_{i+1,i} = \frac{k}{\Delta x} \qquad i = 1, 2, \ldots 9$$

The ten-component column vector $\underline{F}$, represents the forcing function in the problem. It is given by:

$$\underline{F} = \begin{Bmatrix} 200\,000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

where, $f_1 = h\, T_{AW}$

The ten-component volume vector $\underline{T}$ represents the temperature at the ten nodes at any instant of time. The vector $\dot{T}$ is the time derivative of T.

From Appendix F, the analytic solution to the initial value problem (A-1) is given by:

$$\underline{T} = \underline{T}_{\infty} + M^{-1/2}\, V \exp(\Lambda t)\, V^T\, M^{1/2}\, (\underline{T}_{init} - \underline{T}_{\infty}) \qquad \text{Eq. (A-2)}$$

where,

$\underline{T}_\infty$ = the steady-state solution to (A-1)

Λ = a diagonal matrix formed with the eigenvalues λ_i of matrix A

V = a matrix of eigenvectors of matrix A

and,

A = defined by $A = M^{-1/2}\, BM^{-1/2}$

Specifically, these quantities are given by:

$$\underline{T}_\infty = \begin{Bmatrix} 200 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 200 \end{Bmatrix}$$

$$\Lambda = \begin{bmatrix}
-7.48 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -67.34 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -195.27 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & -352.85 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -553.36 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -764.06 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -959.34 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1113.2 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1200.5 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1463.2
\end{bmatrix}$$

$$V = \begin{bmatrix}
-0.049 & -0.133 & 0.190 & 0.221 & -0.235 & -0.233 & -0.214 & -0.166 & 0.067 & -0.841 \\
-0.138 & -0.357 & 0.455 & 0.444 & -0.361 & -0.244 & -0.126 & -0.038 & 0.002 & 0.493 \\
-0.203 & -0.445 & 0.363 & 0.058 & 0.269 & 0.457 & 0.451 & 0.299 & -0.099 & -0.204 \\
-0.263 & -0.435 & 0.049 & -0.396 & 0.408 & 0.006 & -0.399 & -0.462 & 0.192 & 0.085 \\
-0.316 & -0.328 & -0.295 & -0.388 & -0.198 & -0.460 & 0.014 & 0.475 & -0.278 & -0.035 \\
-0.362 & -0.148 & -0.459 & 0.071 & -0.442 & 0.234 & 0.382 & -0.332 & 0.353 & 0.015 \\
-0.399 & 0.065 & -0.342 & 0.448 & 0.121 & 0.338 & -0.460 & 0.081 & -0.415 & -0.006 \\
-0.426 & 0.263 & -0.017 & 0.303 & 0.463 & -0.410 & 0.154 & 0.196 & 0.461 & 0.003 \\
-0.442 & 0.403 & 0.319 & -0.194 & -0.041 & -0.124 & 0.280 & -0.409 & -0.489 & -0.001 \\
-0.317 & 0.321 & 0.325 & -0.329 & -0.333 & 0.336 & -0.340 & 0.346 & 0.352 & 0.001
\end{bmatrix}$$

where the columns of V are eigenvectors of matrix A.

For computational purposes the solution (A-2) is best written as:

$$\underline{T} = \underline{T}_{\infty} + C\,\underline{P}(t) \qquad \text{Eq. (A-3)}$$

where the matrix C is given by

$$C = M^{-1/2}\,V\,D$$

Here D is a diagonal matrix constructed from the components of the column vector $\underline{R}$, defined by:

$$\underline{R} = V^{T}\,M^{1/2}\,(\underline{T}_{init} - \underline{T}_{\infty})$$

That is, the diagonal elements of D are given by:

$$d_{ii} = r_i$$

The time dependent column vector $\underline{P}$ is given by:

$$\underline{P}(t) = \begin{Bmatrix} \exp(-7.48t) \\ \exp(-67.34t) \\ \exp(-185.3t) \\ \exp(-352.9t) \\ \exp(-553.4t) \\ \exp(-764.1t) \\ \exp(-959.3t) \\ \exp(-1113.t) \\ \exp(-1201.t) \\ \exp(-1463.t) \end{Bmatrix}$$

That is, the i^{th} component of $\underline{P}$ is given by:

$$P_i = \exp(\lambda_i t)$$

The matrix C for the particular problem being considered is given by:

$$C = \begin{bmatrix} -19.47 & -16.04 & -11.80 & -8.42 & -6.03 & -4.32 & -2.90 & -1.50 & -0.23 & -29.29 \\ -38.70 & -30.29 & -20.00 & -11.93 & -6.56 & -3.19 & -1.21 & -0.25 & -0.00 & 12.13 \\ -56.98 & -37.82 & -15.96 & -1.55 & 4.89 & 5.98 & 4.32 & 1.91 & 0.24 & -5.03 \\ -73.85 & -36.94 & -2.17 & 10.63 & 7.41 & 0.07 & -3.82 & -2.96 & -0.46 & 2.08 \\ -88.89 & -27.85 & 12.95 & 10.44 & -3.60 & -6.02 & 0.14 & 3.03 & 0.67 & -0.86 \\ -101.74 & -12.58 & 20.15 & -1.91 & -8.03 & 3.07 & 3.66 & -2.12 & -0.85 & 0.36 \\ -112.08 & 5.50 & 15.03 & -12.04 & 2.21 & 4.42 & -4.41 & 0.52 & 1.00 & -0.15 \\ -119.66 & 22.34 & 0.73 & -8.15 & 8.42 & -5.37 & 1.48 & 1.25 & -1.11 & 0.06 \\ -124.28 & 34.23 & -14.03 & 5.23 & -0.74 & -1.62 & 2.68 & -2.62 & 1.17 & -0.03 \\ -125.83 & 38.51 & -20.20 & 12.52 & -8.55 & 6.21 & -4.61 & 3.13 & -1.20 & 0.02 \end{bmatrix}$$

Returning to Eq. (A-3) it can be seen that the temperature at the i^{th} node is given by:

$$T_i = T_{\infty i} + \sum_{j=1}^{N} C_{ij} \exp(\lambda_j t) \qquad \text{Eq. (A-4)}$$

where N equals the number of nodes (10 in this particular case).

For example, the temperature at node 1 is given by:

$$T_1 = 200 - 19.47 \exp(-7.48t) - 16.04 \exp(-67.34t) - 11.80 \exp(-185.35t) - \ldots \qquad \text{Eq. (A-5)}$$

while the temperature at node 10 is:

$$T_{10} = 200 - 125.8 \exp(-7.48t) + 38.51 \exp(-67.34t) - 20.20 \exp(-185.35t) + \ldots \qquad \text{Eq. (A-6)}$$

An examination of the arguments of the exponential functions shows that the leading terms in the finite series dominate the sum.

Two questions naturally arise: (1) how does the HAN solution given in Eq. (A-4) compare, both in form and accuracy, with the exact analytical solution; and (2) what loss in accuracy is incurred by retaining only the dominant E & E's in the solution (i.e., the first few terms in Equation (A-4). These questions are examined in the following discussions.

The analytical solution to the problem being considered is given by the solution to the partial differential equation:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

subject to the boundary conditions,

$$-k \frac{\partial T}{\partial x} (0,t) = h \left[T_{AW} - T (0, t) \right]$$

and

$$\frac{\partial T}{\partial x} (L,t) = 0$$

with initial condition,

$$T (x,0) = T_{init}$$

The solution to this boundary value problem can be obtained by separation of variables in conjunction with Fourier series expansions or simply by looking up the solution to a similar problem in Carslaw and Jaeger's, "Conduction of Heat in Solids" on page 122. In either case, the solution is given by:

$$T (x,t) = T_{AW} + (T_{init} - T_{AW}) \sum_{j=1}^{\infty} \frac{2H \sec (\mu_j)}{H(H+1)+\mu_j^2} \cos \left[\mu_j (1-\frac{x}{L})\right] \exp (-\mu_j^2 \alpha t/L^2)$$

Eq. (A-7)

where the μ_j satisfy the equation

$$\mu_j \tan \mu_j = H \text{ (see Table 1 of Carslaw and Jaeger, p. 491)}$$

and $H = \frac{hL}{k}$

Eq. (A-7) shows that for a given value of x the solution is the form

$$T_i(t) = T_{AW} + \sum_{j=1}^{\infty} a_{ij} \exp(b_j t) \qquad \text{Eq. (A-8)}$$

where the i subscript is used to connote that the temperature is for a particular value of x and where

$$a_{ij} = (T_{init} - T_{AW}) \frac{2H \sec(\mu_j)}{H(H+1) + \mu_j^2} \cos\left[\mu_j \left(1 - \frac{x_i}{L}\right)\right]$$

and

$$b_j = -\mu_j^2 \frac{\alpha}{L^2}$$

Eq. (A-8) is identical in form with Eq. (A-4) except that in Eq. (A-8) the series is an infinite one rather than a finite one. (Note that T_∞ in Eq. (A-4) represents the steady-state solution at node i, which is in fact T_{AW}).

It is instructive to compare the C_{ij}'s with the a_{ij}'s and the λ_j's with the b_j's. These comparisons are shown in the following tables for the two extreme locations for x, i.e., x = 0 and x = L corresponding respectively to i = 1 and i = 10.

	i = 1	x = 0	i = 10	x = L
j	C_{1j}	a_{1j}	C_{10j}	a_{10j}
1	-19.47	-19.56	-125.83	-125.98
2	-16.04	-16.63	38.51	38.80
3	-11.80	-12.72	-20.20	-20.40
4	-8.42	-9.33	12.52	12.46
5	-6.03	-6.84	-8.55	-8.28
6	-4.32	-5.10	6.21	5.83

The λ_j's and b_j's are independent of x. They are as follows:

j	λ_j	b_j
1	-7.48	-7.49
2	-67.34	-68.19
3	-185.27	-192.90
4	-352.85	-385.43
5	-553.36	-648.50
6	-764.06	-986.60

The comparison between the sets of constants is very good. This is to be expected since it is known that as the number of nodal points N approaches infinity the C_{ij}'s will approach the a_{ij}'s and the λ_j's will approach the b_j's, and, therefore, in the limit, the HAN solution approaches the exact solution. But the really pertinent question here is: how accurate is the 10-point HAN solution? Figures A-2 and A-3 address this question. They show comparisons of the HAN solution with the exact solution at the front and rear faces of the slab. If all ten terms (i.e., eigenvalues and eigenvectors) of the HAN solution are used, the temperatures calculated are virtually identical to those of the exact solution. As a matter of fact for times in excess of 0.05 hr, the first exponential term in the HAN solution compares with the exact solution to within 0.06°K. It is very interesting to observe that the shorter the time period of interest the greater the number of terms required to achieve a given accuracy. An alternative point of view to this observation is perhaps more significant. Namely, for a given number of terms or eigenvalues there is a minimum time period to achieve a certain accuracy. It may be noted that $(\Delta x)^2/\alpha$ can be considered a characteristic time for the transient response of nodal elements. For this example, $(\Delta x)^2/\alpha \simeq 0.003$ hr, and it can be seen that the time required for accuracy with only a few dominant eigenvalues is of the order of three times this value. A further note is that the typical explicit finite algorithm has a maximum time step (for the problem being studied it is 0.00165 hr). This maximum time step arises from stability requirements, and is not related to the accuracy considerations for the HAN method.

Two things need emphasizing at this juncture. First, with the HAN method considerable computer machine time can be saved by calculating only those E & E's that are "significant". This was noted very aptly by Maise and Rossi in NASA CR-2435 and used by them in the CAPE code for the indirect heat transfer problem. As regards the direct heat transfer problem being solved here, Figures A-2 and A-3 show that for a time period of interest greater than 0.01 hr, it would be a waste of machine time to compute the fourth through tenth E & E's since they would have an insignificant effect on the computed temperature.

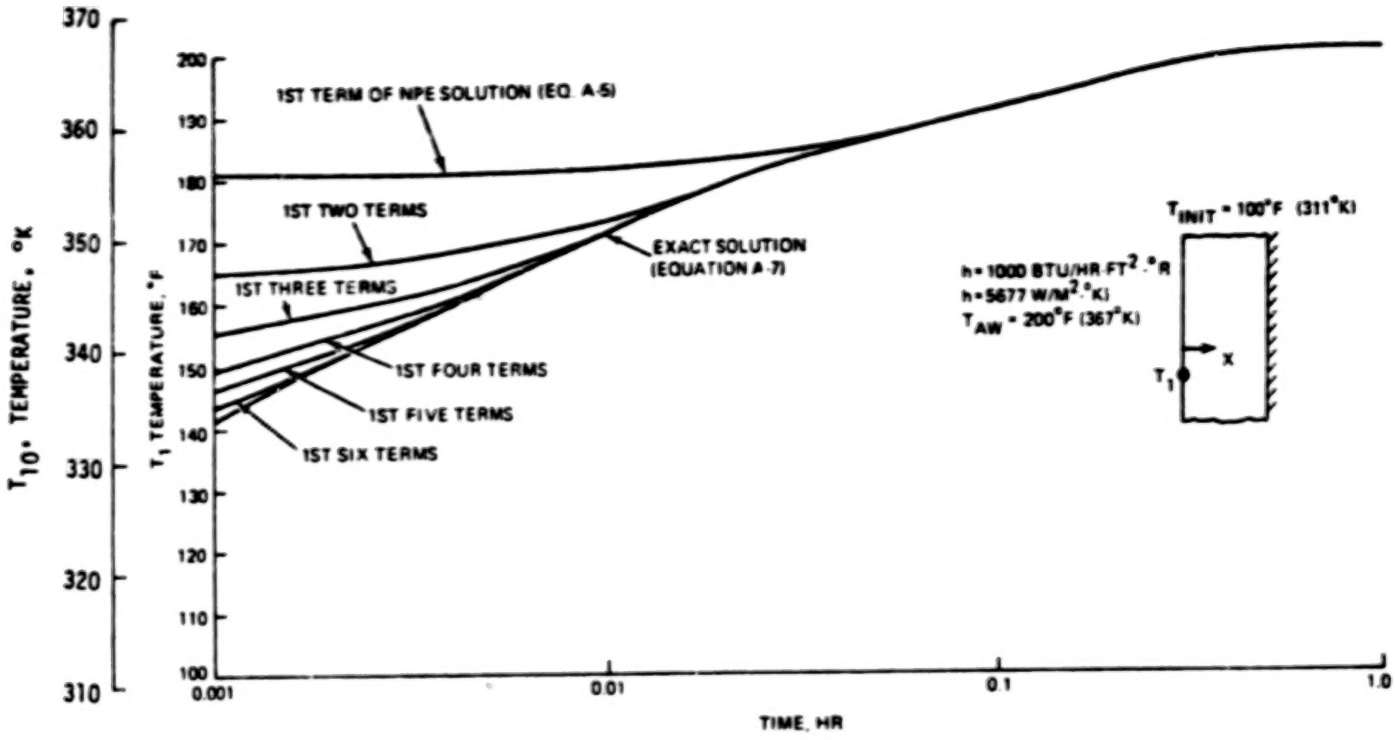


FIG. A-2 COMPARISON OF NODAL POINT EIGENVALUE (NPE) SOLUTION AND THE EXACT SOLUTION AT X = 0

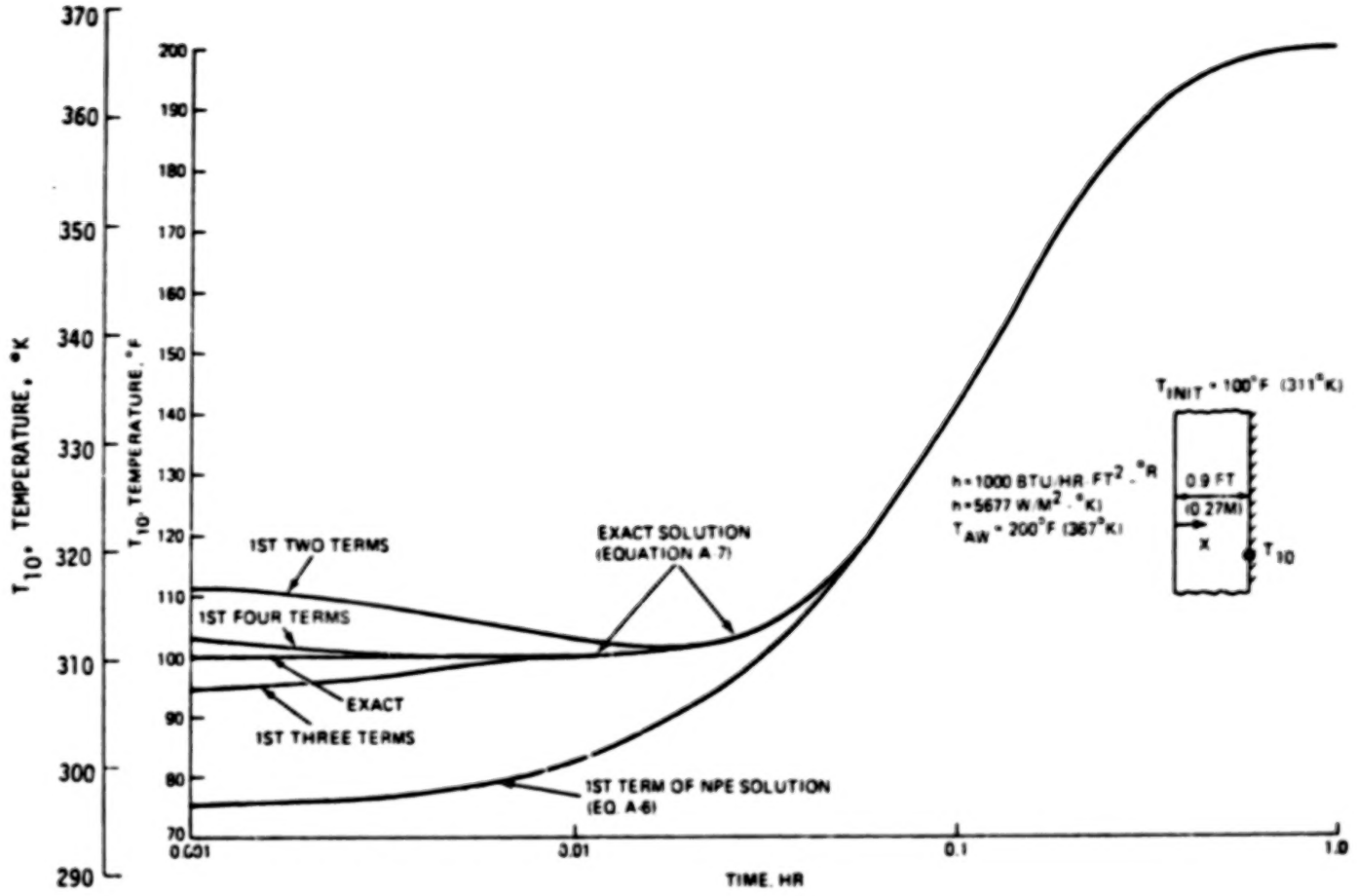


FIG. A-3 COMPARISON OF NODAL POINT EIGENVALUE SOLUTION AND THE EXACT SOLUTION AT X = 0.9

The second item that needs emphasizing is that because the boundary conditions on the problem studied herein were constant with respect to time, it was necessary to obtain the E & E's only once. Time varying boundary conditions will require periodic revision of the E & E's to reflect the changes in matrix A. In essence, problems involving time varying boundary conditions are solved by dividing the time interval into M subintervals within which the boundary conditions will be constant. Thus there will be M subproblems with the initial conditions of one problem being the final conditions of the previous problem. Obviously, frequent revisions imply a small time period and hence more E & E's to achieve a given accuracy. Some reflection on all of this reveals that the optimal solution (viz., minimum machine time for a given accuracy level or maximum accuracy level for a given machine time) will be achieved by proper selection of the number of E & E revisions (or time steps) and the number of E & E's to be calculated. The following table illustrates this point. In Table A-1, M represents the optimal number of E & E's revisions and NE represents the optimal number of E & E's.

Table A-1. Mix of Constant Machine Time Solutions

Time Period Between E & E Revision	No. of E & E Revisions	No. of E & E's	Remarks
Long	$<M$	$>NE$	With long time period, do not need this many E & E's. Furthermore, the piecewise constant representation of boundary condition is poor
Optimal	M	NE	Number of E & E's calculated consonant with the time period. Good piecewise constant representation of time varying boundary conditions
Short	$>M$	$<NE$	Number of E & E's insufficient to provide good accuracy for this short time period. Piecewise constant representation of boundary condition is excellent

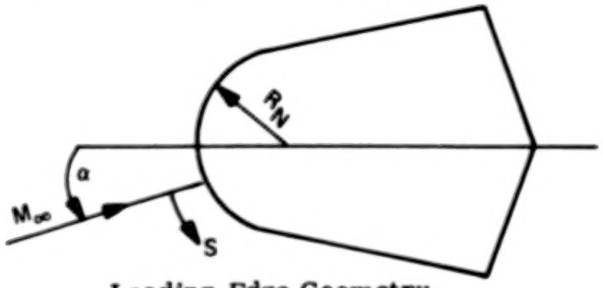
The "optimal" values for M and NE depend somewhat on the particular flight trajectory, geometry, dimensions and materials being analyzed. For the missile and Hypersonic Research Aircraft problems that were analyzed during the development of the CAVE code, NE should be approximately 3 to 5, and M should be approximately twice the number of flight trajectory points input to the code.

In summary, this appendix has presented the details of the HAN technique applied to a one-dimensional conduction problem. The HAN solution gave a near perfect comparison with the exact analytical solution of the same problem. It was seen that the HAN solution has a minimum time interval for a given accuracy level and number of E & E's. Furthermore, we saw that there is an optimal number of time subdivisions and E & E's for a specified accuracy level or machine time.

APPENDIX B

AERODYNAMIC HEATING EQUATIONS*

Incorporated within the CAVE code are equations to predict the aerodynamic heat transfer that will occur at the surfaces of the leading edge and flat plate geometries. This appendix presents the equations that are used in subroutines OVLY20, LEES1 and FLATH. The equations are valid up to a Mach number of six or so, where real gas effects become important. Ambient pressure and temperature as functions of altitude are calculated in subroutine ATMOS based on the 1962 U.S. Standard Atmosphere. The range of altitude is from sea level to 47 350 m.



Leading Edge Geometry

Aerodynamic heating of the leading-edge geometry is handled in two steps: (1) the convective heat transfer coefficient at the stagnation point is computed in subroutine OVLY20 and; (2) the ratios of the local convective values to the stagnation point value are calculated in subroutine LEES1 for all the surface nodes.

The user can flag CAVE to use either a turbulent or laminar flow correlation for the stagnation point coefficient.

For turbulent flow, CAVE uses Beckwith and Gallagher's equation (B-1):

$$= \frac{k_\infty}{2R_N} Re_\infty^{4/5} Pr^{1/3} \left(0.0228 \frac{\mu_W}{\mu_0} \frac{T_\infty}{T_W} \frac{p_e}{p_\infty} \right)^{4/5} (\sin \Lambda_{eff})^{3/5} \left[\frac{49}{376} \frac{\mu_0}{\mu_\infty} \cos \Lambda_{eff} \left(\frac{2R_N}{V_\infty} \frac{dV}{dS} \right)_0 \right]^{1/5}$$

Eq. (B-1)

*The reference list is included at the end of this appendix; Table B-1 gives the nomenclature.

where according to reference (B-2):

$$\left(\frac{2R_N}{V_\infty}\frac{dV}{dS}\right)_o = \frac{2a_o}{a_\infty M_{eff_\infty}}\left[\frac{2}{\gamma}\left(1-\frac{p_\infty}{p_{t_o}}\right)\right]^{1/2} \qquad \text{Eq. (B-2)}$$

The effective sweep angle Λ_{eff} is given in Reference B-3.

$$\Lambda_{eff} = \sin^{-1}\left[\sin\Lambda\cos\alpha + \sin\alpha\cos\Lambda\sin\Psi\right] \qquad \text{Eq. (B-3)}$$

with α being the angle of attack, Ψ the dihedral angle and Λ the actual sweep angle.

For laminar flow with a freestream Mach number less than two, CAVE uses the modified Lees equation for the stagnation point:

$$h_o = 0.5\,\frac{C_p}{Pr^{2/3}}\left[\rho_e^*\left(\frac{\mu_e^*}{3600\,g_c}\right)V_o'\right]^{1/2} \qquad \text{Eq. (B-4)}$$

where the * quantities are evaluated at

$$T^* = 0.5\,(T_W + T_o)$$

The velocity gradient term V_o' is computed using:

$$V_o' = \frac{V_\infty}{R_N}\left[\frac{\rho_\infty}{\rho_o}\left(1-\frac{\rho_\infty}{\rho_o}\right)\right]^{1/2} \qquad \text{Eq. (B-5)}$$

For freestream Mach numbers greater than two, CAVE uses Detra, Kemp and Ridell's equation (Reference B-4) modified for effective sweep in accordance with References B-5 and B-6, to give the laminar heat transfer coefficient at the stagnation point:

$$h_o = \frac{2.21}{(T_{t_o}-540)}\left(\frac{\rho_\infty}{R_N}\right)^{1/2}\left(\frac{V_\infty}{1000}\right)^{3.15}\cos^{1.5}\Lambda_{eff} \qquad \text{Eq. (B-6)}$$

where Λ_{eff} is the effective sweep angle given by Eq. (B-3).

With the stagnation point coefficient h_o thus established, the distribution of convective coefficients around the leading edge is computed using Lees' formula (Reference B-7), with assumptions of cold wall and $\mu \sim T$:

$$h = h_o F V_\infty^{1/2} / \quad (dV_e/dS)_o$$

where

$$F \equiv \frac{p}{p_{t_o}} \frac{V_e}{V_\infty} \left[2 \int_o^S \frac{p}{p_{t_o}} \frac{V_e}{V_\infty} dS \right]^{-1/2}$$

Maise and Rossi (Reference B-8) used this formula in subroutine LEES to obtain the distribution of h around the leading edge. The stagnation point value h_o was an unknown in their problem and was found by the CAPE code given the temperature history of the body. Basically, this same subroutine (LEES1) has been incorporated in the CAVE code.

To evaluate F quantitatively, the local flow conditions around the leading edge are required. These conditions are found using the modified Newtonian law in the subsonic region surrounding the stagnation point and using Prandtl-Meyer expansions downstream of the sonic point locations. Specifically, in the subsonic region the local conditions will be predicted by the following equations:

$$p_o = p_{t_\infty} \left[\frac{2\gamma M_\infty^2 \sin^2\phi - (\gamma - 1)}{\gamma + 1} \right] \left[\frac{2}{(\gamma - 1) M_\infty^2 \sin^2\phi + 2} \right]^{\frac{\gamma}{\gamma - 1}}$$

where $\phi = 90° - \Lambda_{eff}$

The local pressure distribution is predicted according to Reference B-1:

$$p_e = p_\infty + (p_o - p_\infty) \cos^2\theta$$

where θ is the angle measured from the stagnation point.

$$p_{t_o} = p_{t_\infty} \left[\frac{(\gamma+1)\, M_\infty^2 \sin^2\phi}{(\gamma-1)\, M_\infty^2 \sin^2\phi + 2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_\infty^2 \sin^2\phi - (\gamma-1)} \right]^{\frac{\gamma}{\gamma-1}}$$

$$M_e = \left\{ \frac{2}{\gamma-1} \left[\left(\frac{p_{t_o}}{p_e} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{1/2}$$

$$T_e = T_{t_\infty} / \left(1 + \frac{\gamma-1}{2} M_e^2\right)$$

$$a_e = \sqrt{\gamma g_c R T_e}$$

$$V_e = M_e a_e$$

$$\rho_e = p_e / R T_e$$

In the supersonic region downstream of the sonic points the local Mach number, M_e, is obtained via the Prandtl-Meyer turning angle ν found from the following equations:

$$\nu_2 = \nu_1 + \left[\theta_2 - \theta_1\right]$$

where

$$\nu(M) = \sqrt{(\gamma+1/\gamma-1)}\ \tan^{-1} \sqrt{(\gamma-1/\gamma+1)\ (M_e^2-1)}\ - \tan^{-1} \sqrt{M_e^2-1}$$

and

$\left[\theta_2 - \theta_1\right]$ represents the change in flow angle.

With the local Mach number determined, the other pertinent flow properties are established using the following equations:

$$p_e = p_{t_o} \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-\gamma/\gamma-1}$$

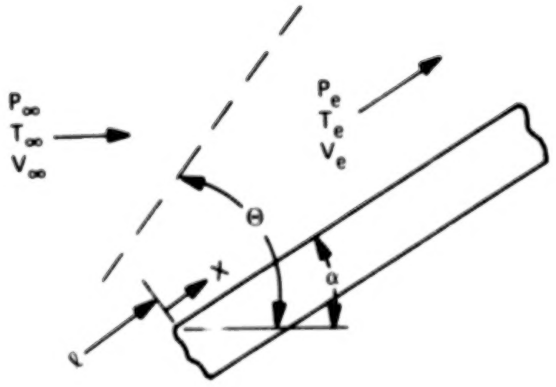
$$T_e = T_{t_o} / \left(1 + \frac{\gamma - 1}{2} M_e^2\right)$$

$$a_e = \sqrt{\gamma g_c RT_e}$$

$$V_e = M_e a_e$$

$$\rho_e = p_e / RT_e$$

Flat Plate Geometry



For flat plate geometries Eckert's reference enthalpy method (Reference B-8) is employed to predict the heat transfer rate, i.e.,

$$h = \frac{0.332 \rho^* V_e C_p}{Re^{*1/2} Pr^{2/3}} \qquad \text{(laminar flow)} \qquad \text{Eq. (B-7)}$$

and

$$h = \frac{0.0296 \rho^* V_e C_p}{Re^{*1/2} Pr^{2/3}} \qquad \text{(turbulent flow)} \qquad \text{Eq. (B-8)}$$

The Reynolds number is based on the boundary layer length ℓ .

The subroutine FLATH calculates the local flow values to be used in the above equations based on oblique shock wave theory. That is, the shock angle θ is found from solving the following cubic equation:

$$Z^3 + bZ^2 + cZ + d = 0$$

where,

$$Z = \sin^2\theta$$

$$b = -\frac{M_\infty^2 + 2}{M_\infty^2} - \gamma \sin^2\alpha$$

$$c = \frac{2M_\infty^2 + 1}{M_\infty^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M_\infty^2}\right] \sin^2\alpha$$

$$d = -\frac{\cos^2\alpha}{M_\infty^4}$$

With the appropriate θ selected for a weak shock, the other flow properties are calculated via the following equations:

$$p_e = p_\infty \left[\frac{2\gamma M_\infty^2 \sin^2\theta - (\gamma-1)}{\gamma+1}\right]$$

$$T_e = T_\infty \frac{\left[2\gamma M_\infty^2 \sin^2\theta - (\gamma-1)\right]\left[(\gamma-1) M_\infty^2 \sin^2\theta + 2\right]}{(\gamma+1)^2 M_\infty^2 \sin^2\theta}$$

$$V_e = V_\infty \left[1 - \frac{4 (M_\infty^2 \sin^2\theta - 1)(\gamma M_\infty^2 \sin^2\theta + 1)}{(\gamma+1)^2 M_\infty^4 \sin^2\theta}\right]^{1/2}$$

$$\rho_e = p_e / RT_e$$

The subroutine TRANS establishes the laminar and turbulent regions using the boundary layer transition criterion shown in Figure B-1 (Reference B-9). It is recognized that this criterion is not the final word on simple boundary layer transition criteria but rather representative of the best presently available. In light of this, the code is written so that as newer criteria are evolved they may be readily incorporated into subroutine TRANS. For that matter, the entire aerodynamic heating portion of the code is written so that as the need arises to use more specialized equations, they may be easily substituted for those presented herein.

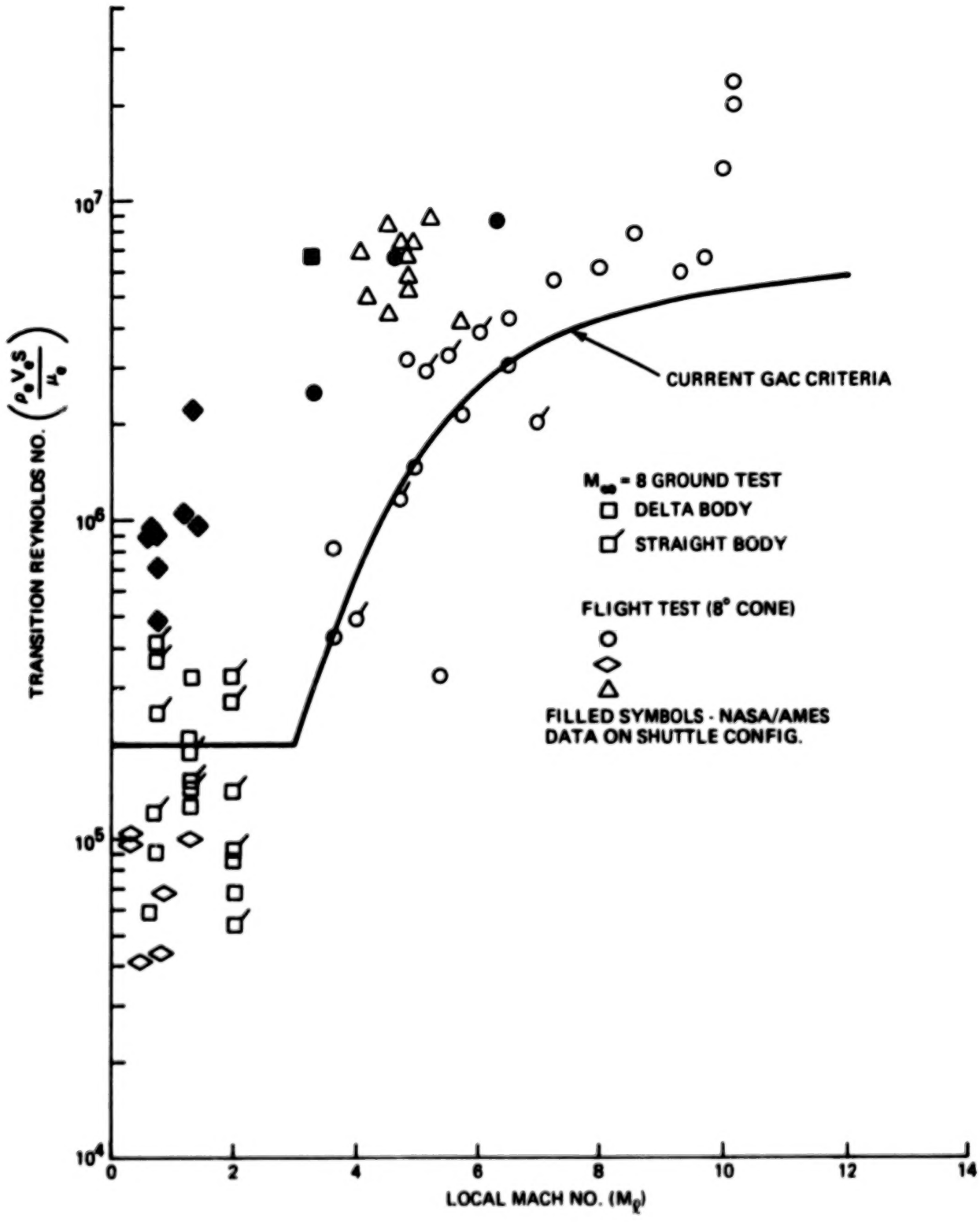


FIG. B-1 TRANSITION CRITERION

TABLE B-1. NOMENCLATURE AND UNITS

Symbol		Units
a	Speed of sound	ft/sec
C_p	Specific heat at constant pressure	Btu/lbm-°R
d	Hydraulic diameter	ft
g_c	Newton constant	32.17 ft-lbm/lbf-sec^2
h	Convective heat transfer coefficient	Btu/ft^2-sec-°R
k	Thermal conductivity	Btu-ft/ft^2-sec-°R
ℓ	Effective boundary layer length	ft
M	Mach number	dimensionless
p	Pressure	lbf/ft^2
Pr	Prandtl number	dimensionless
q	Heat flux	Btu/ft^2-sec
r	Recovery factor	dimensionless
R	Gas constant	lbf-ft/lbm-°R
Re	Reynolds number	dimensionless
R_N	Nose radius	ft
S	Distance along surface	ft
T	Temperature	°R
V	Velocity	ft/sec
α	Angle of attack	degrees
γ	Ratio of specific heats	dimensionless
θ	Angular position from stagnation point or oblique shock angle	degrees
Λ	Wing sweep angle	degrees
ρ	Mass density	lbm/cu-ft
μ	Viscosity	lbm/ft-hr
Subscripts		
AW	Adiabatic wall	
e	Property at edge of boundary layer	
eff	Effective	

(Continued)

TABLE B-1. NOMENCLATURE AND UNITS (Cont'd)

Symbol		Units
Subscripts (Cont'd)		
o	Stagnation point	
r	Recovery	
t	Total condition (i.e., condition that would exist if fluid brought to rest isentropically)	
W	Wall	
∞	Free stream	
Superscripts		
*	indicates fluid property evaluated at temperature T^* given by $T^* = T_e + 0.5(T_W - T_e) + 0.22\,(T_r - T_e)$	

Appendix B References

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B-3. G. Maise and M. J. Rossi, "Lateral Conduction Effects on Heat-Transfer Data Obtained with the Phase-Change Paint Technique," NASA CR-2435, August 1974, p. 25.

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B-7. W. H. Dorrance, "Viscous Hypersonic Flow," McGraw-Hill Co., 1962.

B-8. G. Maise and M. J. Rossi, op. cit., p. 22.

B-9. E. R. Eckert, "Survey on Heat Transfer at High Speeds," WADC Technical Report 54-70, April 1954.

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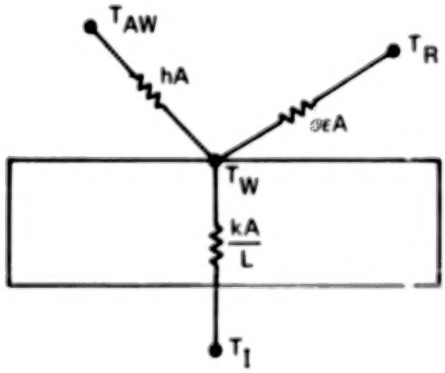
APPENDIX C

LINEARIZATION OF RADIATION COUPLING

Leading edges, cooled panels and slab geometries may involve radiation heat transfer at the surface. Typically the direction of heat transfer is away from the body, serving to cool it. An exact treatment of radiation within each time step precludes an eigenvalue-eigenvector solution which depends on the problem being a linear one. Thus radiation heat transfer is given a linear representation with each time step in such a way that for the step size approaching zero the exact solution is produced. This appendix presents the linearization within each time step. It is the common one of modifying the convection coupling to account for the radiation, which typically amounts to a reduction in the convection coupling.

Consider a node diagram for a surface node of a body.



where

hA = convection coupling

$\sigma\epsilon A$ = radiation coupling

$\frac{kA}{L}$ = conduction coupling

T_{AW} = adiabatic wall temperature of fluid (more properly referred to as recovery temperature of fluid)

T_R = background radiation temperature (usually taken as 0°K)

T_W = temperature of surface node

T_I = temperature of interior node

The heat transfer into the surface due to convection and radiation is

$$Q_{in} = h\,A\,(T_{AW} - T_W) + \sigma\epsilon A\,(T_R^4 - T_W^4) \qquad \text{Eq. (C-1)}$$

This may be rewritten as:

$$\dot{Q}_{in} = (h + h_R)\,A\,(T_{AW} - T_W) \qquad \text{Eq. (C-2)}$$

where

$$h_R = \frac{\sigma\epsilon\,(T_R^4 - T_W^4)}{(T_{AW} - T_W)} \qquad \text{Eq. (C-3)}$$

Eq. (C-2) can be rewritten into the form used with CAVE:

$$\dot{Q}_{in} = h_{eff}\;A\,(T_{AW} - T_W) \qquad \text{Eq. (C-4)}$$

where

$$h_{eff} = h + h_R \qquad \text{Eq. (C-5)}$$

Since T_R is usually taken to be 0°K, h_R will be negative and, therefore, the effective convective coupling h_{eff} will be less than the actual convective coupling h. The term h_R is a linearized radiative coupling and it is in essence a correction factor to the convection coupling. Frequently, this correction factor amounts to less than 5% of the convection coupling and is, therefore, of no significant consequence. On the other hand it is possible, particularly for high altitude trajectories, to have a relatively large correction factor, so large that the effective convective coupling is actually negative. A similar situation can occur late in a re-entry vehicle trajectory following peak heating. A negative h does not appear to pose any particular difficulty to the matrix routines within CAVE. However, a difficulty has been observed when h_{eff} approaches zero (i.e., radiation coupling and convection coupling essentially equal). In these situations there are ill-conditioned matrices involved in the problem solution and the cumulative effect of arithmetic roundoff becomes a serious matter in the eigenvector-eigenvalue iteration procedure within subroutine IJEN.

This difficulty has been observed on occasion with the manifestation of a failure within IJEN to obtain estimates for the eigenvalues and eigenvectors that were within tolerance. (A statement to this effect is printed out by CAVE.) However, the temperatures calculated appeared correct. This may be attributed to the eigenvector values being reasonable although not within tolerance and furthermore not being of great significance since the body is not changing very much in temperature due to the very small coupling between it and its environment. Double precision arithmetic would help to alleviate this "ill-conditioned" difficulty.

One final item to be mentioned is that in calculating h_R from Eq. (C-3), CAVE uses the temperature of the surface nodes at the beginning of time subinterval (end of previous time subinterval). Therefore, in problems where radiation plays an important role and where the surface temperatures are varying rapidly with time, the user should use small time subintervals (perhaps one fifth of the trajectory table time intervals).

APPENDIX D

PROGRAMMER ORIENTED DOCUMENTATION OF THE CODE

This appendix presents the details of the CAVE organization and structure. A simplified logic flow diagram of CAVE is given in Figure D-1.

CAVE is organized in a main program with 36 subroutines. The list of subroutines is given in Table D-1 together with the function of each subroutine and the calling routine. Figure D-2 presents the organization of CAVE in terms of the more important subroutine calls.

Many subroutines are either identical to, or modified versions of, subroutines used in the CAPE code (NASA CR-2435), which was developed by Maise and Rossi to solve the inverse problem of finding the convective heat transfer coefficient given temperature history information. A modified version of a CAPE subroutine has a slight change in name to avoid any possibility of the wrong subroutine being used, as would be the case if both the CAVE and CAPE codes were resident on the same disc. To illustrate typical name changes, consider the CAPE subroutines PCP, HETRA and SLAB; in CAVE these subroutines in modified form are referred to as PCP4, HETRA1 and SLAB2, with no significance attributed to the integer value.

Flow charts or descriptions for each subroutine are given in Figures D-3 through D-26. For convenience and completeness, they are given for subroutines that are identical to the CAPE routines reported in NASA CR-2435 as well as the new subroutines.

Sets of input data for check cases were presented in Sections 3 through 5 where the standard output for these were described. For detailed output that maps the iterations and eigenvalues, the flags LTE and MON must be set equal to +6 in statement cards within SIZE2 (normal values are -6). The detailed output refers to the iterations performed in the subroutine DESDA1 and the subroutines that are called from it. The detailed output can be used to establish whether sufficient eigenvalues have been selected by the user since the detailed output shows the contribution of each term in the equation (Refer to Appendix A, Eq. (A-8) with the number of

nodes N replaced by the number of dominant eigenvalues NE):

$$T_i = T_{\infty_i} + \sum_{j=1}^{NE} C_{ij} \exp(\lambda_j t)$$

Each succeeding contribution should be smaller than the previous one, with the contribution of the last term being small in comparison with the accuracy desired for the temperatures. If this is not the case, NE should be increased and the problem rerun at the expense of increased computer time.

CAVE prints out self-explanatory diagnostic messages for some of the errors that may be caused by faulty input data preparation. A diagnostic message "FAILURE IN IJEN" signifies a failure of the Jennings algorithm to converge within the maximum number of iterations in the subroutine IJEN. This message has been observed on rare occasions as a result of a matrix being ill-conditioned at a particular time in the flight trajectory where the radiation-convection coupling is approaching zero (refer to Appendix C). In these situations the temperatures have appeared to be correct and the message has been ignored. If corrective action should be necessary it could include the following: increasing the maximum number of iterations as given by NIJ in subroutine SIZE2 from its present value of 20; increasing the tolerance on the convergence test as given by TIJ in subroutine SIZE2 from its present value of 0.1; revising the time steps used to avoid this troublesome point in the flight trajectory; and as a last resort utilizing double precision arithmetic.

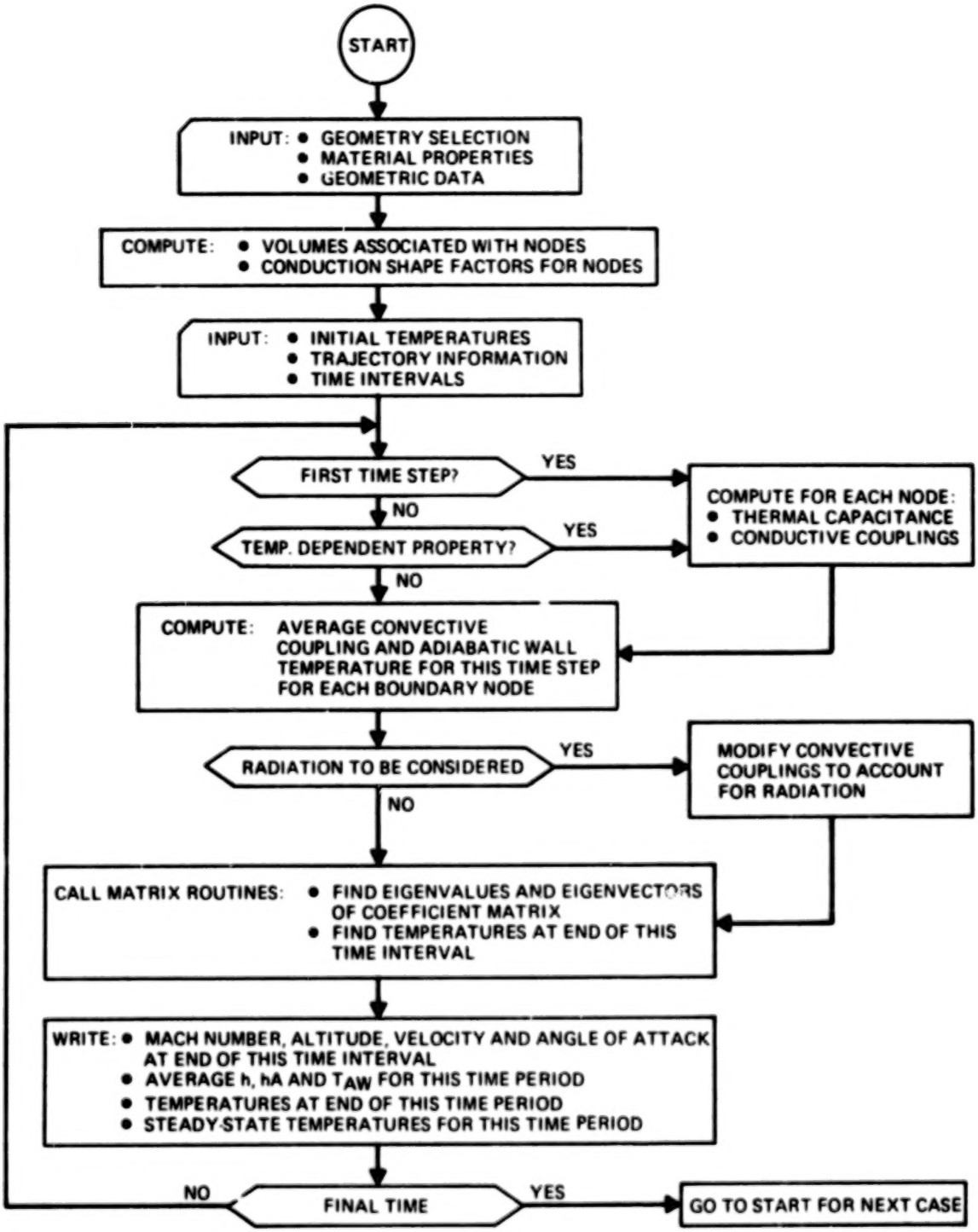

START
INPUT: • GEOMETRY SELECTION
• MATERIAL PROPERTIES
• GEOMETRIC DATA
COMPUTE: • VOLUMES ASSOCIATED WITH NODES
• CONDUCTION SHAPE FACTORS FOR NODES
INPUT: • INITIAL TEMPERATURES
• TRAJECTORY INFORMATION
• TIME INTERVALS
FIRST TIME STEP?
YES
NO
COMPUTE FOR EACH NODE:
• THERMAL CAPACITANCE
• CONDUCTIVE COUPLINGS
TEMP. DEPENDENT PROPERTY?
YES
NO
COMPUTE: AVERAGE CONVECTIVE COUPLING AND ADIABATIC WALL TEMPERATURE FOR THIS TIME STEP FOR EACH BOUNDARY NODE
RADIATION TO BE CONSIDERED
YES
NO
MODIFY CONVECTIVE COUPLINGS TO ACCOUNT FOR RADIATION
CALL MATRIX ROUTINES: • FIND EIGENVALUES AND EIGENVECTORS OF COEFFICIENT MATRIX
• FIND TEMPERATURES AT END OF THIS TIME INTERVAL
WRITE: • MACH NUMBER, ALTITUDE, VELOCITY AND ANGLE OF ATTACK AT END OF THIS TIME INTERVAL
• AVERAGE h, hA AND T_{AW} FOR THIS TIME PERIOD
• TEMPERATURES AT END OF THIS TIME PERIOD
• STEADY-STATE TEMPERATURES FOR THIS TIME PERIOD
NO
FINAL TIME
YES
GO TO START FOR NEXT CASE

FIG. D-1 SIMPLIFIED FLOW OF LOGIC IN CAVE

Table D-1. Subroutines Used in CAVE

Name	Called By	Main Purpose
SIZE2*	CAVE (main prog)	Computes storage locations needed. Compares to number requested
OVLY10*	SIZE2	Sets up arrays for PCP4
PCP4*	OVLY10	Reads and writes property data, controls geometry
SLAB2*	PCP4	Computes volumes and conduction shape factors for slab and cooled panel geometries
LEAD4*	PCP4	Computes volumes and conduction shape factors for leading edge geometries
X24C	PCP4	Computes volumes and conduction shape factors for basic X24C geometries
GEN	PCP4	Reads and writes volumes and conduction shape factors for general geometries
MATOUT	PCP4	Writes material properties
XTABS1	GEN	Reads tabular values of hA and T_{AW} for general geometry problems
OVLY20	SIZE2	Reads initial temperature distribution and flight trajectory. Controls problem solution, steps time, computes average convection couplings for each time step. Writes solution each time step
LINFIT	OVLY20,PROP	Finds value from a table by linear interpolation
ATTAC2*	OVLY20	Finds node number closest to stagnation point and renumbers nodes as required by LEES1
LEES1*	ATTAC2	Computes ratios h/h_{sp} and T_{AW} variation around leading edge problems
NURED	OVLY20	Reads tabular values of hA and T_{AW} as functions of distance and time
DINTK	OVLY20	Finds value from a table using double interpolation
PROP	OVLY20	Computes conduction couplings and mass specific heat product for each element given conduction shape factors and volumes
PRPOUT	OVLY20	Writes node numbers, conductances, capacitances and initial temperatures
XINTP1	OVLY20	Finds values of several dependent variables from a table by linear interpolation on a single independent variable
FLATH	OVLY20	Finds h and T_{AW} for flow over a flat plate
ATMOS	OVLY20,FLATH	Finds atmospheric pressure, temperature and density for given altitude
POLRT	FLATH	Computes the roots of a polynomial

(continued)

Table D-1. Subroutines Used in CAVE (Cont'd)

Name	Called By	Main Purpose
TRANS	FLATH	Determines whether flow over flat plate will be considered laminar or turbulent
DESDA1	OVLY20	Calls eigenvalue and matrix routines. Calculates temperatures
IJEN+	DESDA1	Obtains dominant eigenvectors and eigenvalues of a given matrix (using Jennings method of simultaneous vector iteration)
EIGVC+	DESDA1	Prepares approximate guesses for the eigenvectors to start the Jennings algorithm iteration for the first time step
BFACS+	IJEN	Factorizes a banded positive-definite matrix
BSOLS+	DESDA1, IJEN	Using the factors of a given banded positive-definite matrix A as generated by BFACS solves for X in the system AX = Y
ORNML+	IJEN, DESDA1	Carries out the standard Gram-Schmidt orthonormalization of a group of vectors
HETRA1*	DESDA1	Sets up coefficient matrix (of conductances) in compact form
RVORDR+	IJEN	Reorders estimated eigenvalues according to magnitude
AORDER+	IJEN, RVORDER	Sets up permutation indices needed for ordering the eigenvalues
DISPLA+	(Various)	Prints information, mainly debug special output, in array form
PART+	DESDA1, PCP4	Prints debug output information and intermediate timing of calculation
DATE+	PART	Determines data of run
SWITCH+	DISPLAY	Converts columns of a matrix to rows or visa versa
SCAPR2+	(Various)	Computes scalar product of two vectors

*Modified version of CAPE routine

+ CAPE routine

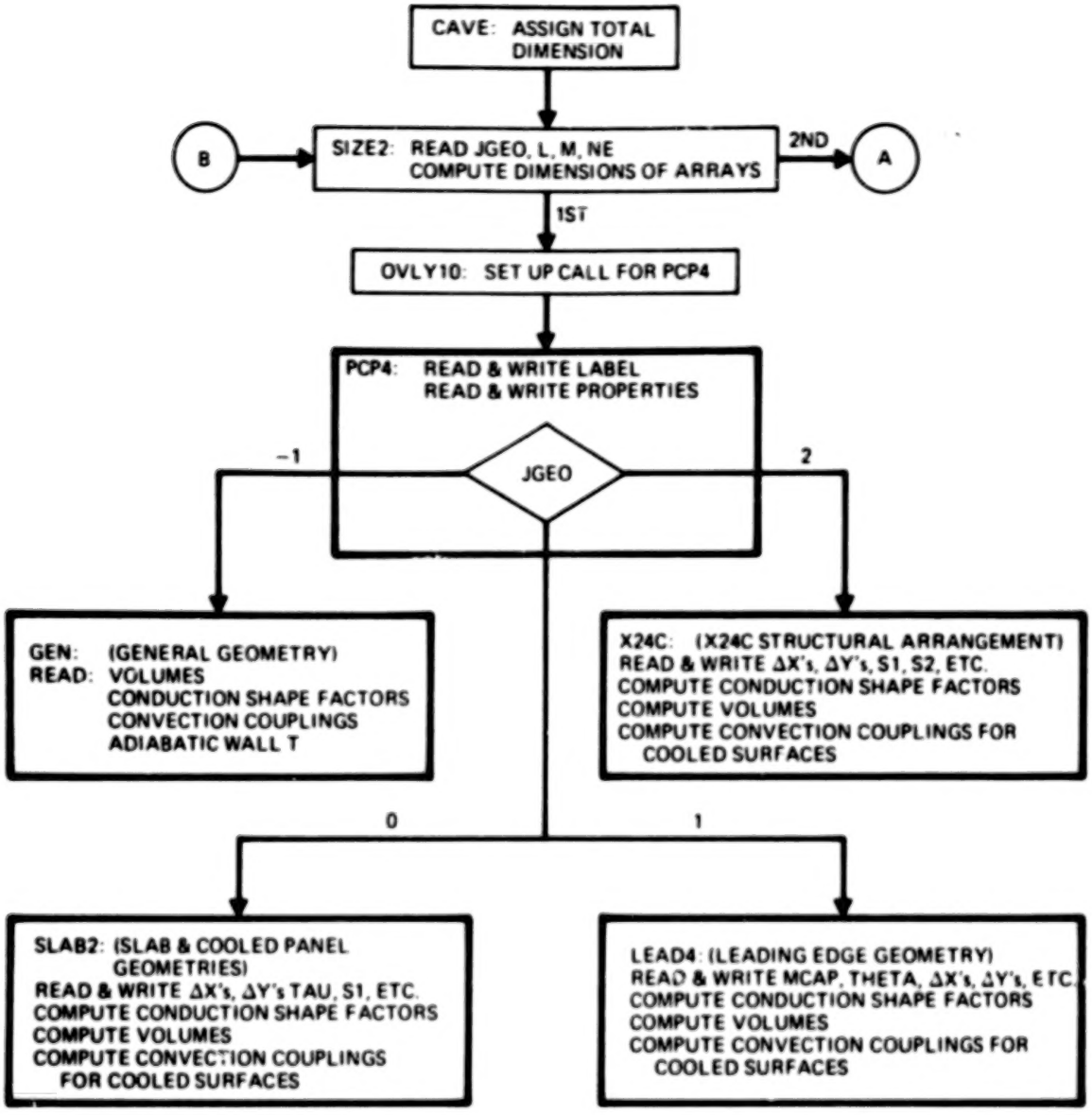


FIG. D-2 ORGANIZATION OF CAVE IN TERMS OF THE MORE IMPORTANT SUBROUTINE CALLS (SHEET 1 OF 2)

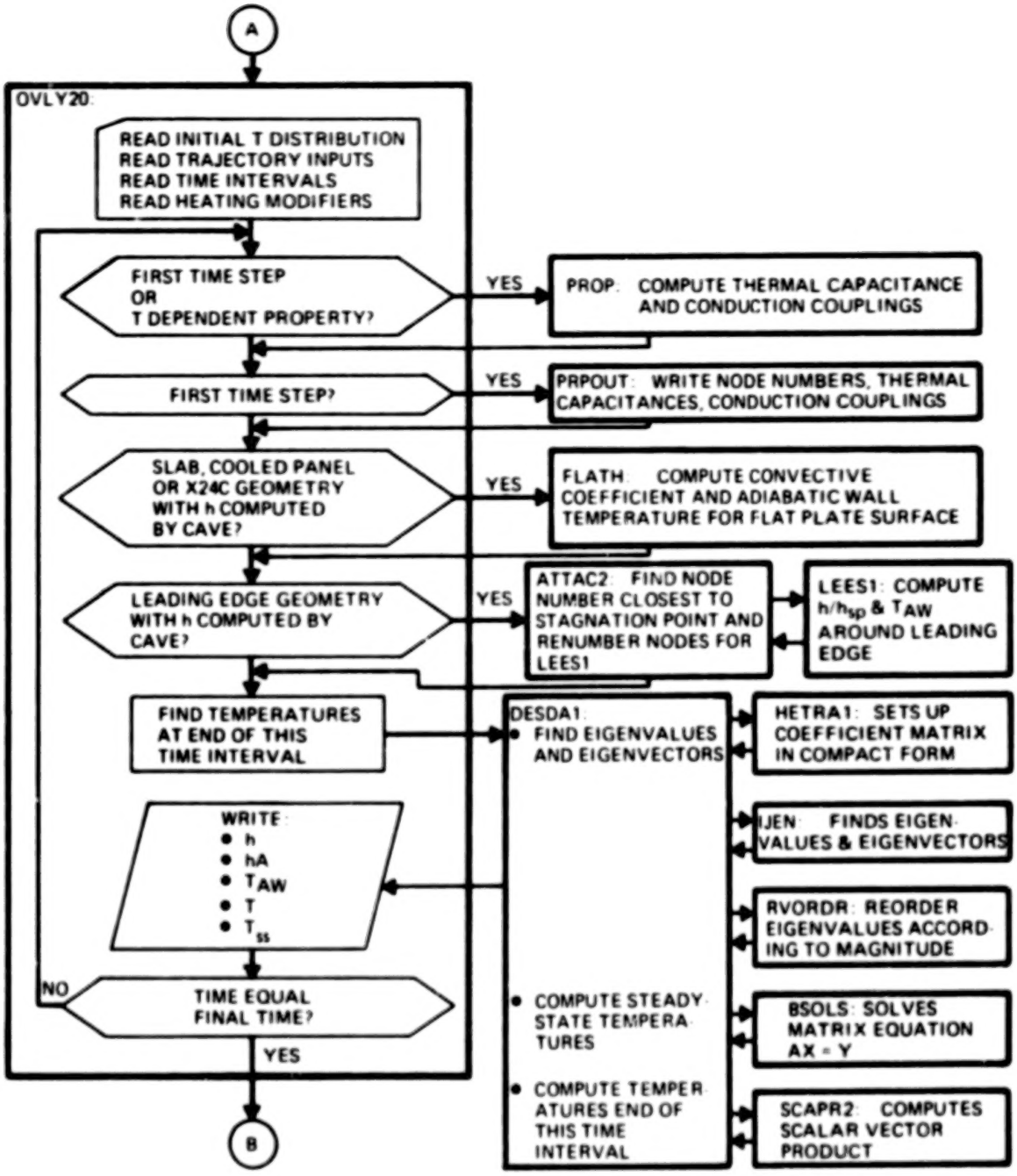

A
OVLY20
READ INITIAL T DISTRIBUTION
READ TRAJECTORY INPUTS
READ TIME INTERVALS
READ HEATING MODIFIERS
FIRST TIME STEP OR T DEPENDENT PROPERTY?
YES
PROP: COMPUTE THERMAL CAPACITANCE AND CONDUCTION COUPLINGS
FIRST TIME STEP?
YES
PRPOUT: WRITE NODE NUMBERS, THERMAL CAPACITANCES, CONDUCTION COUPLINGS
SLAB, COOLED PANEL OR X24C GEOMETRY WITH h COMPUTED BY CAVE?
YES
FLATH: COMPUTE CONVECTIVE COEFFICIENT AND ADIABATIC WALL TEMPERATURE FOR FLAT PLATE SURFACE
LEADING EDGE GEOMETRY WITH h COMPUTED BY CAVE?
YES
ATTAC2: FIND NODE NUMBER CLOSEST TO STAGNATION POINT AND RENUMBER NODES FOR LEES1
LEES1: COMPUTE h/hsp & TAW AROUND LEADING EDGE
FIND TEMPERATURES AT END OF THIS TIME INTERVAL
DESDA1:
• FIND EIGENVALUES AND EIGENVECTORS
HETRA1: SETS UP COEFFICIENT MATRIX IN COMPACT FORM
IJEN: FINDS EIGENVALUES & EIGENVECTORS
RVORDR: REORDER EIGENVALUES ACCORDING TO MAGNITUDE
WRITE
• h
• hA
• TAW
• T
• Tss
NO
TIME EQUAL FINAL TIME?
YES
• COMPUTE STEADY STATE TEMPERATURES
BSOLS: SOLVES MATRIX EQUATION AX = Y
• COMPUTE TEMPERATURES END OF THIS TIME INTERVAL
SCAPR2: COMPUTES SCALAR VECTOR PRODUCT
B

FIG. D-2 ORGANIZATION OF CAVE IN TERMS OF THE MORE IMPORTANT SUBROUTINE CALLS (SHEET 2 OF 2)

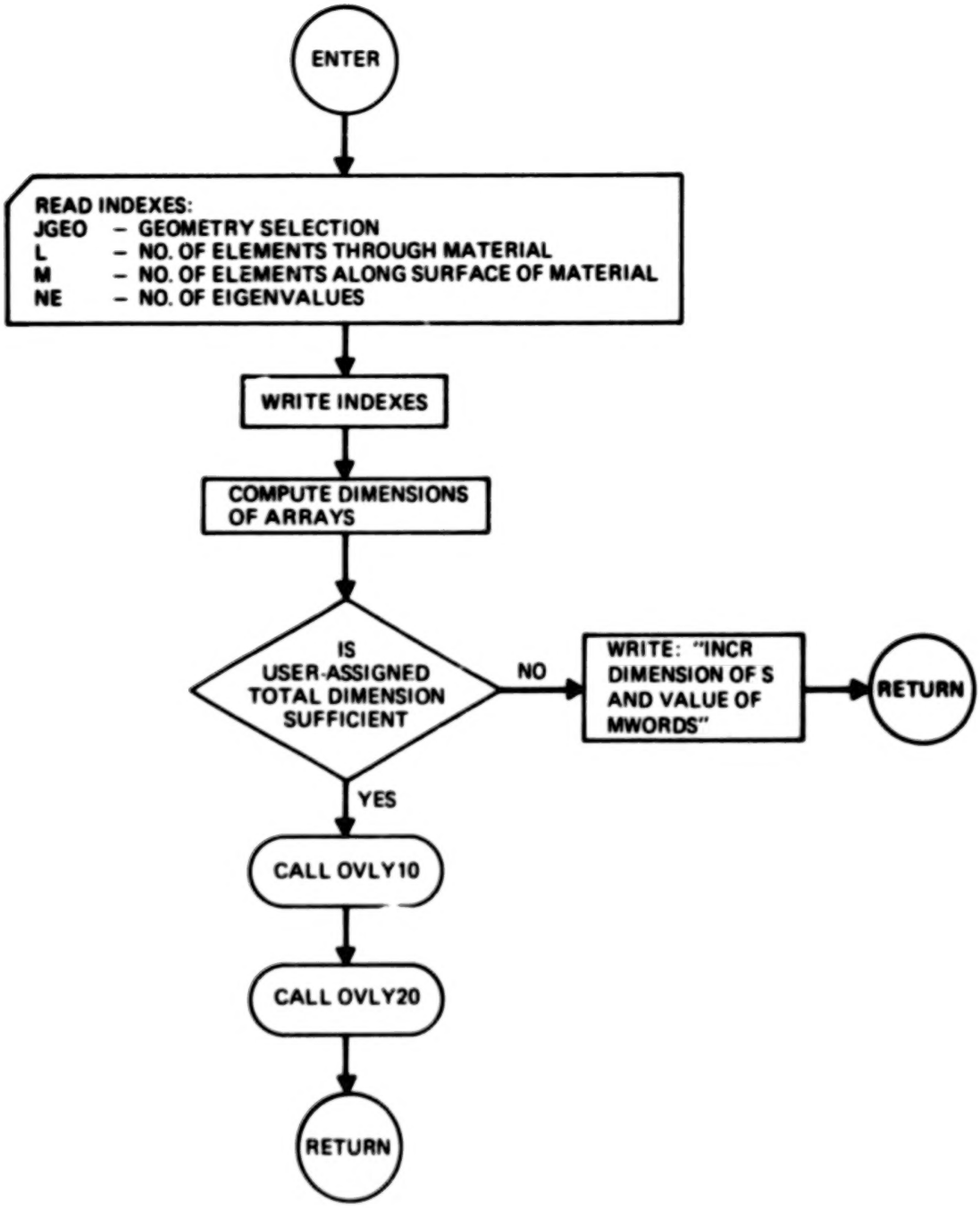


FIG. D-3 SUBROUTINE SIZE2 FLOW CHART

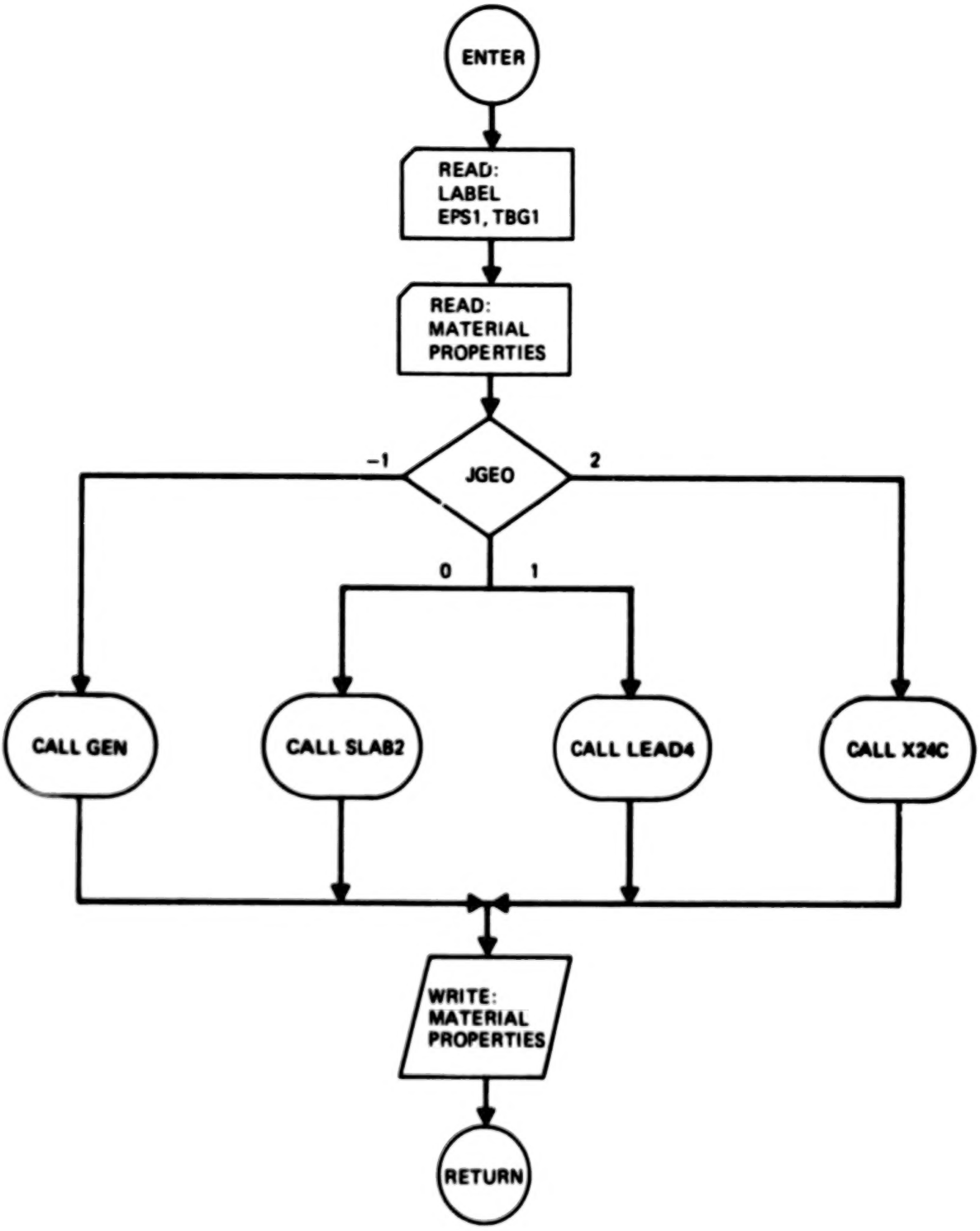


FIG. D-4 SUBROUTINE PCP4 FLOW CHART

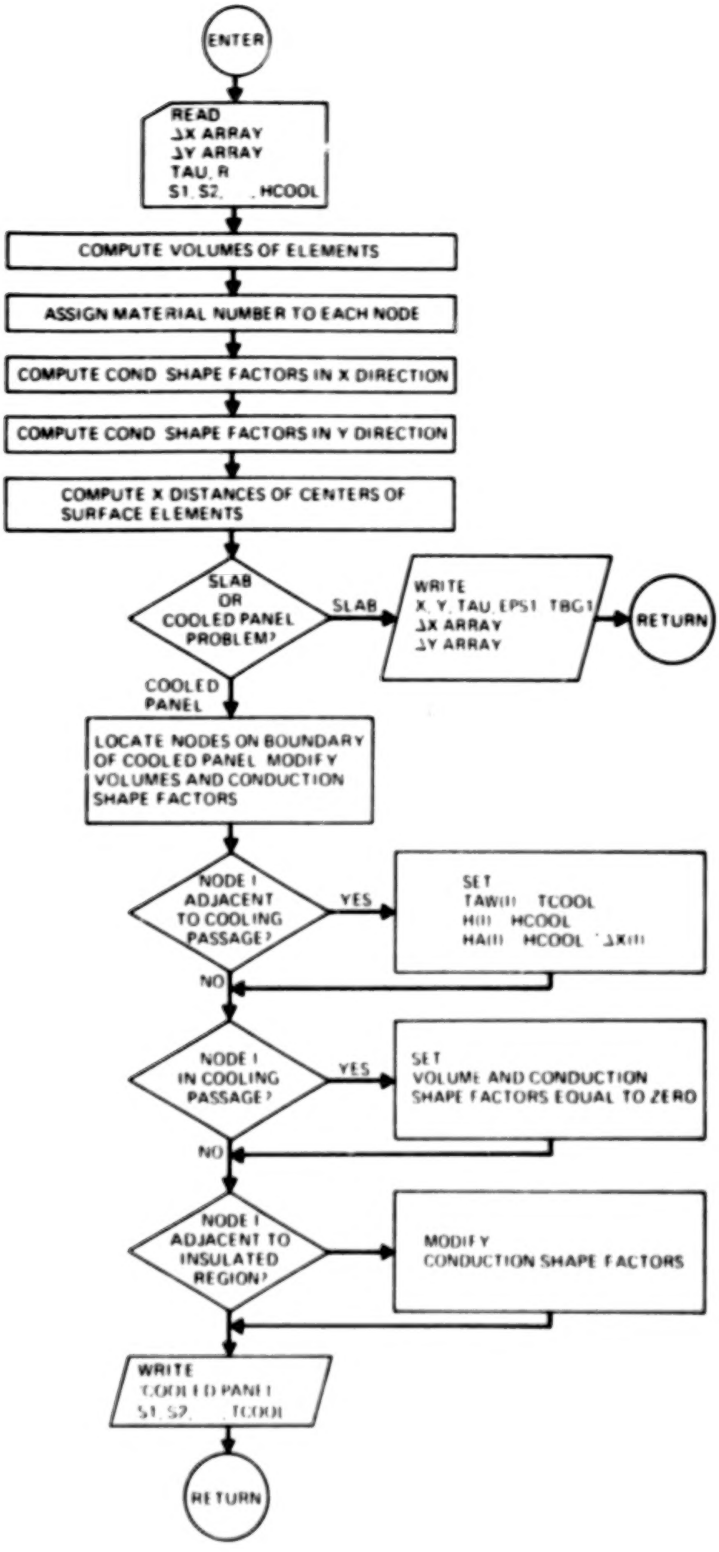


FIG. D-5 SUBROUTINE SLAB2 FLOW CHART

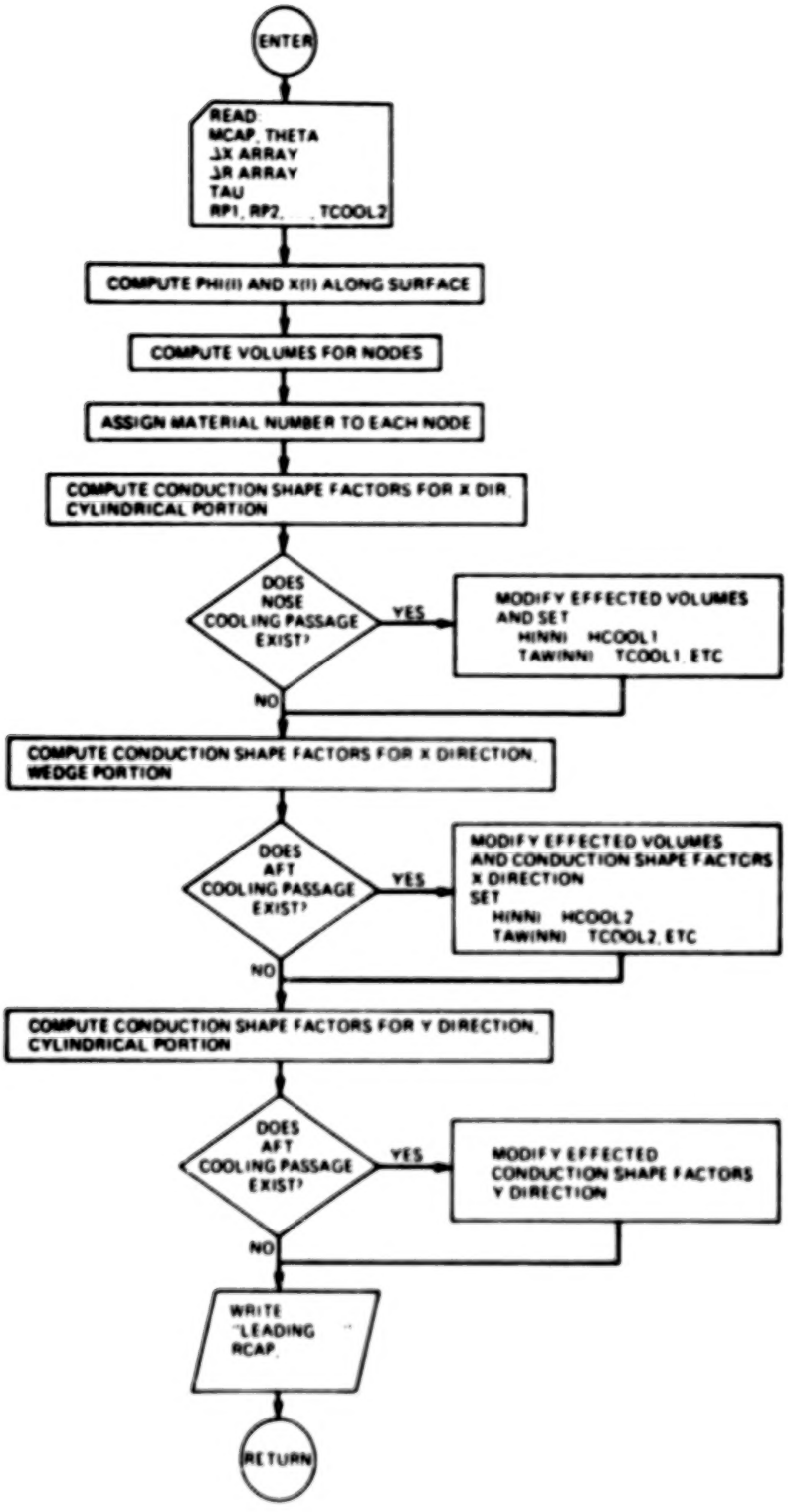


FIG. D-8 SUBROUTINE LEAD4 FLOW CHART

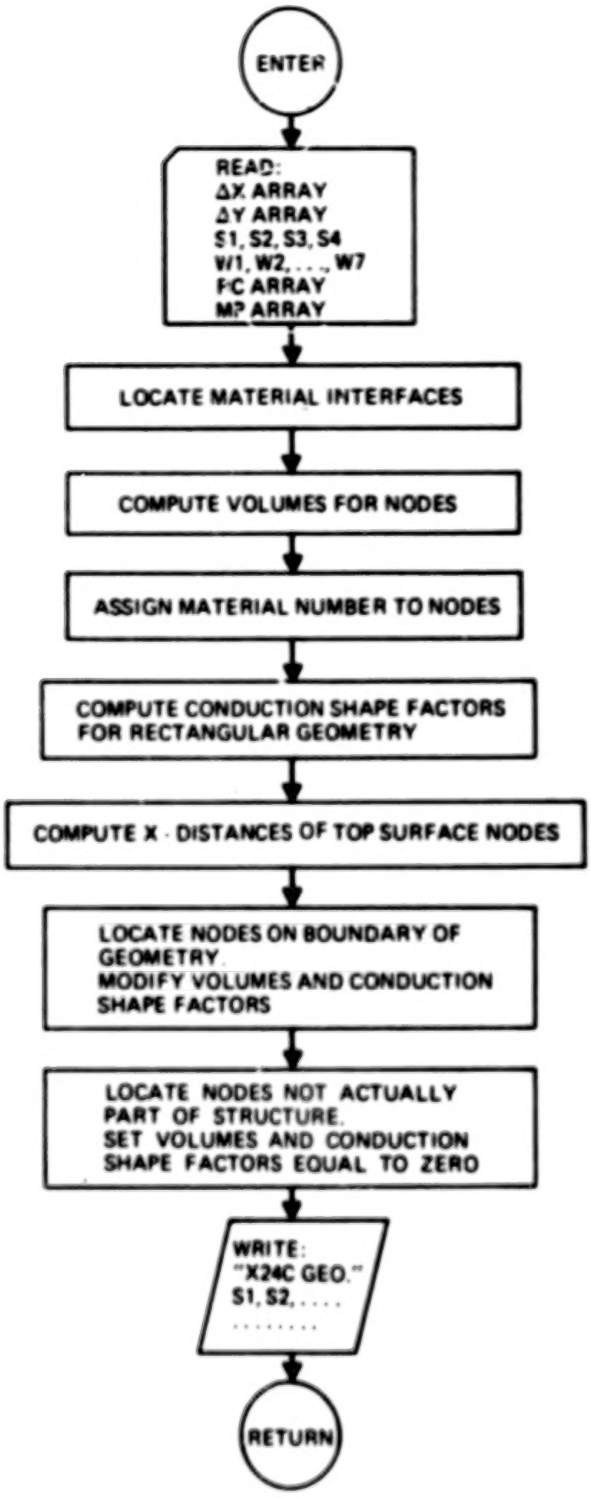
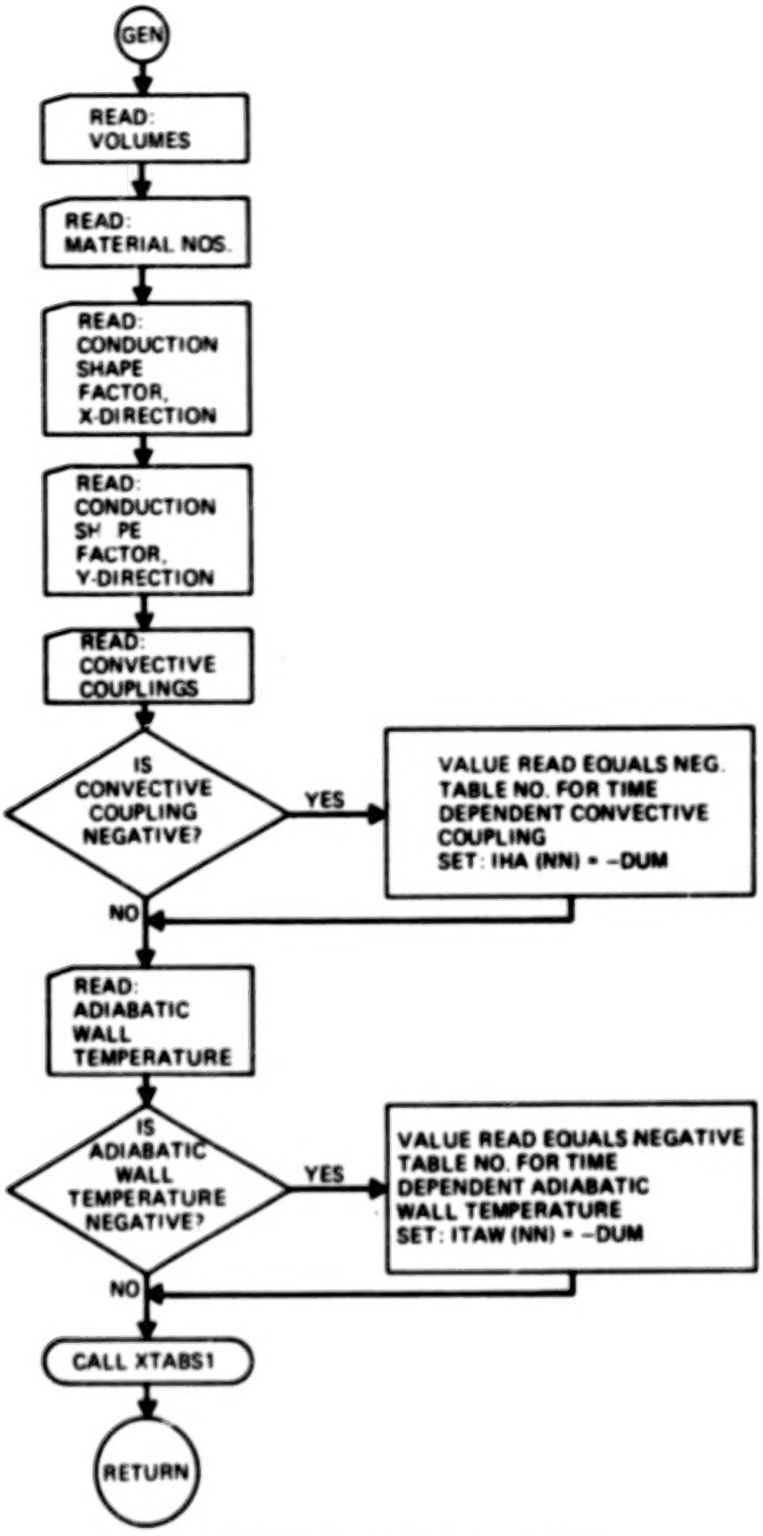

ENTER
READ:
ΔX ARRAY
ΔY ARRAY
S1, S2, S3, S4
W1, W2, . . ., W7
FC ARRAY
MP ARRAY
LOCATE MATERIAL INTERFACES
COMPUTE VOLUMES FOR NODES
ASSIGN MATERIAL NUMBER TO NODES
COMPUTE CONDUCTION SHAPE FACTORS FOR RECTANGULAR GEOMETRY
COMPUTE X - DISTANCES OF TOP SURFACE NODES
LOCATE NODES ON BOUNDARY OF GEOMETRY.
MODIFY VOLUMES AND CONDUCTION SHAPE FACTORS
LOCATE NODES NOT ACTUALLY PART OF STRUCTURE.
SET VOLUMES AND CONDUCTION SHAPE FACTORS EQUAL TO ZERO
WRITE:
"X24C GEO."
S1, S2, ...
. . . . . .
RETURN

FIG. D-7 SUBROUTINE X24C FLOW CHART



IFG. D-8 SUBROUTINE G[illegible]OW CHART

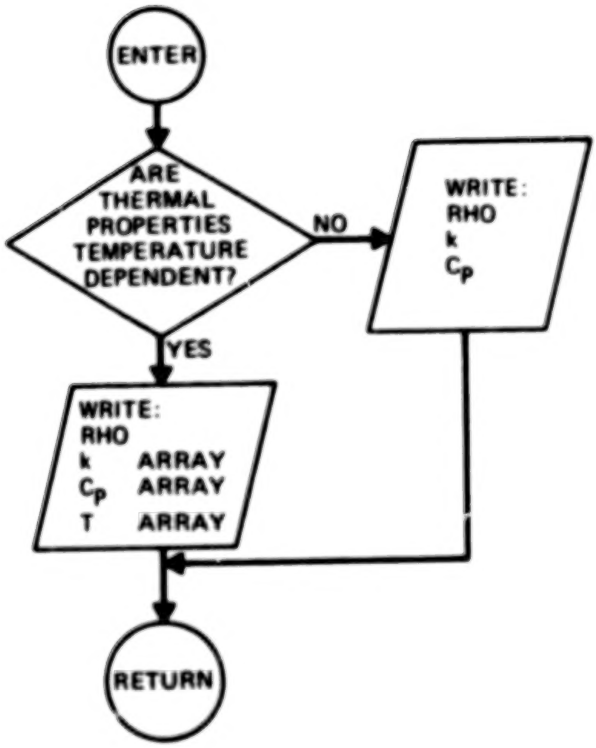


FIG. D-9 SUBROUTINE MATOUT FLOW CHART

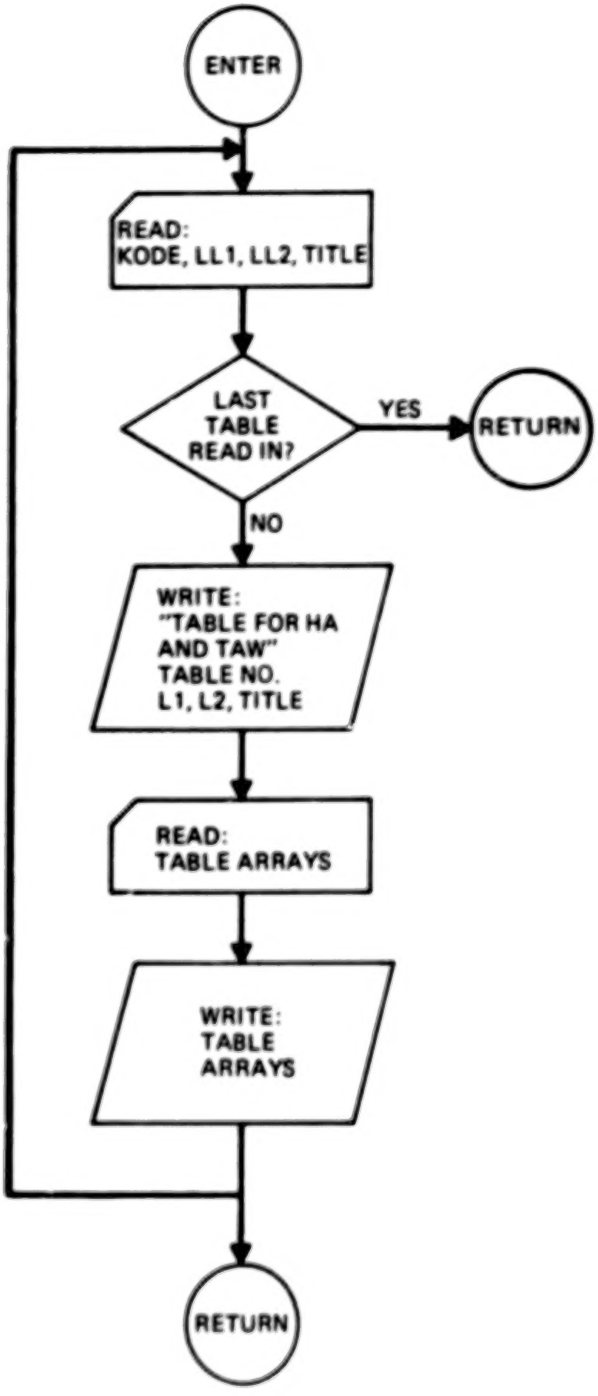


FIG. D-10 SUBROUTINE XTABS1 FLOW CHART

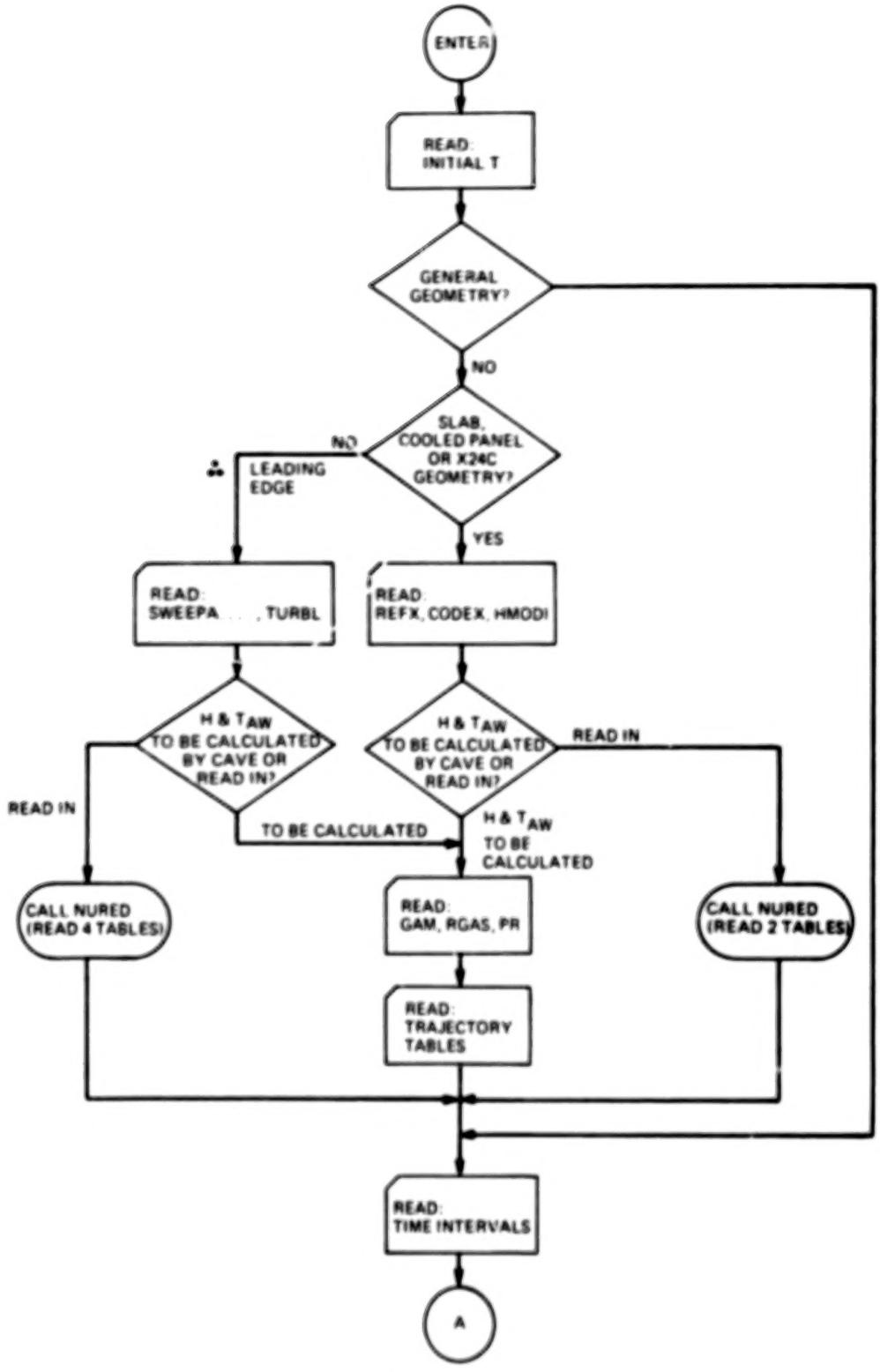


FIG. D-11 SUBROUTINE OVLY20 FLOW CHART (SHEET 1 OF 4)

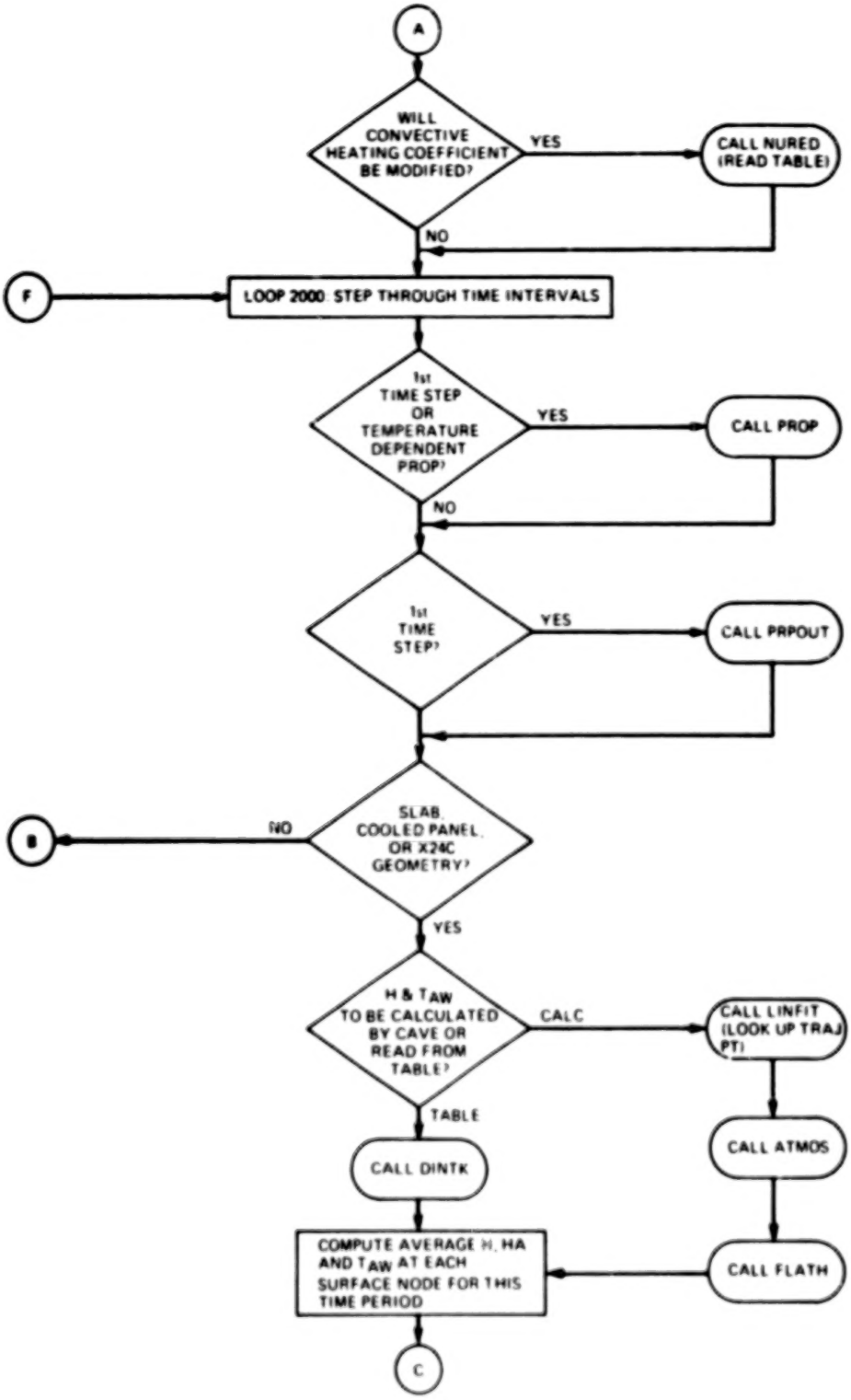


FIG. D-11 SUBROUTINE OVLY20 FLOW CHART (SHEET 2 OF 4)

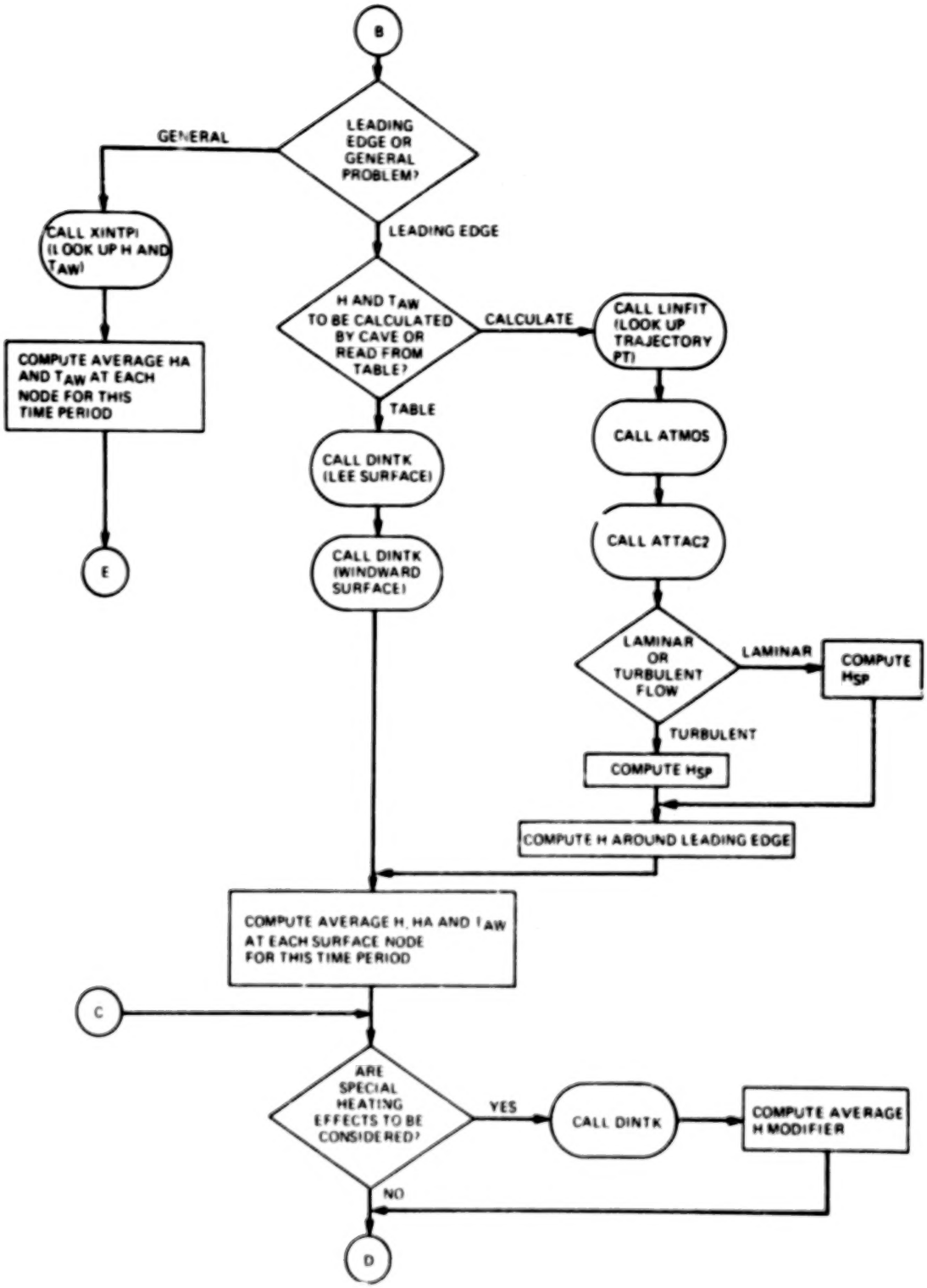


FIG. D-11 SUBROUTINE OVLY20 FLOW CHART (SHEET 3 OF 4)

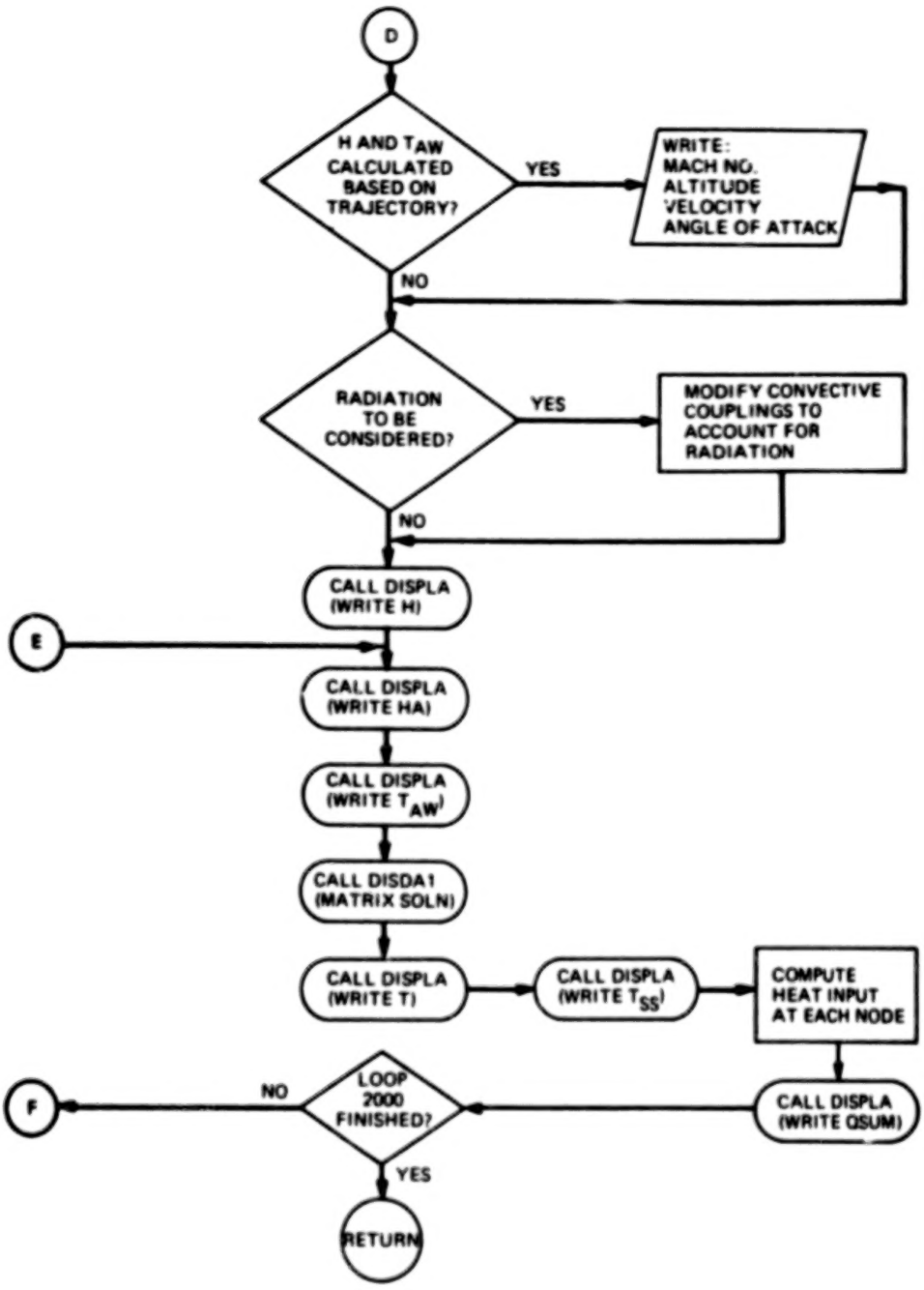
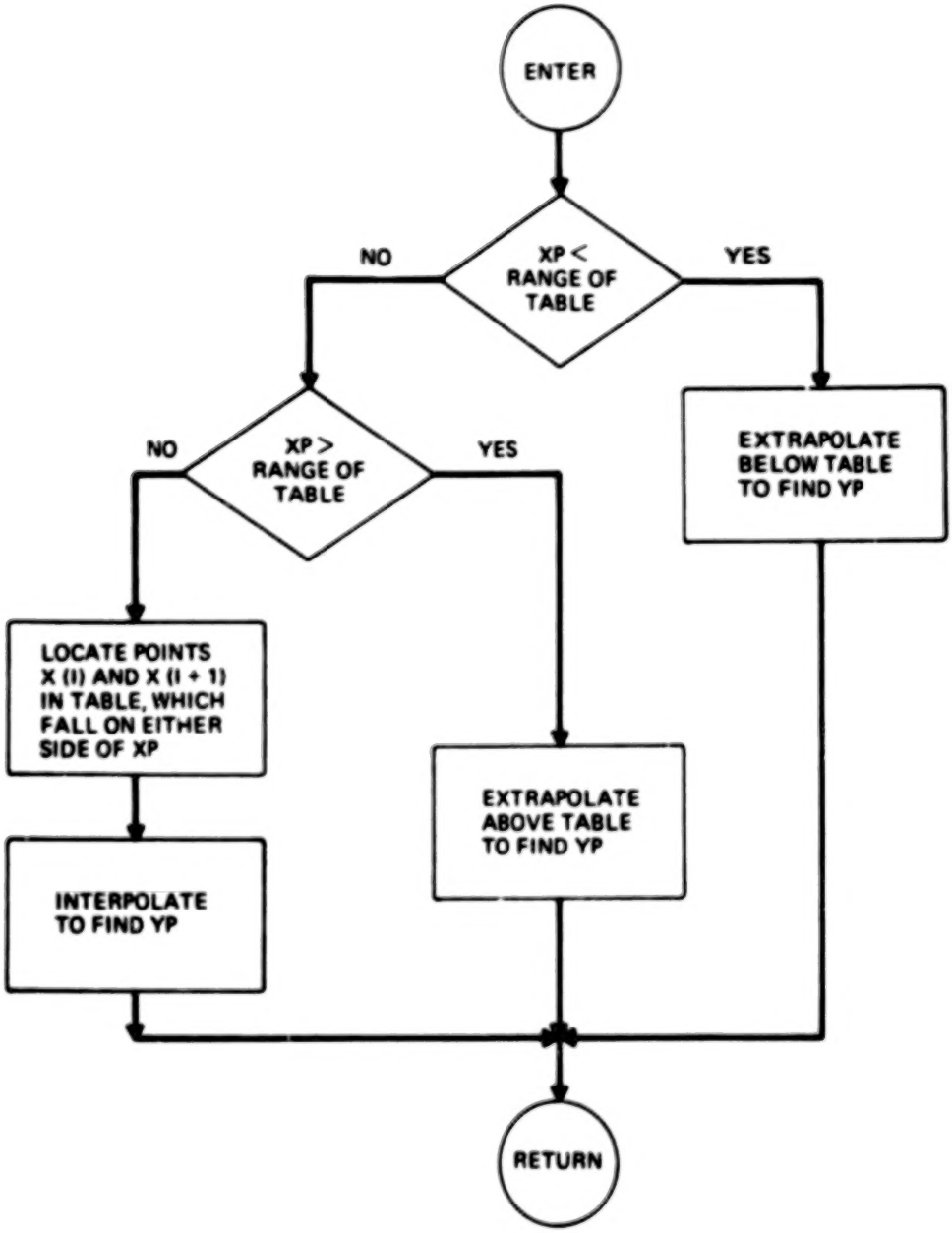


FIG. D-11 SUBROUTINE OVLY20 FLOW CHART (SHEET 4 OF 4)



*(SOURCE: NASA CR-2435)

FIG. D-12 SUBROUTINE LINFIT FLOW CHART*

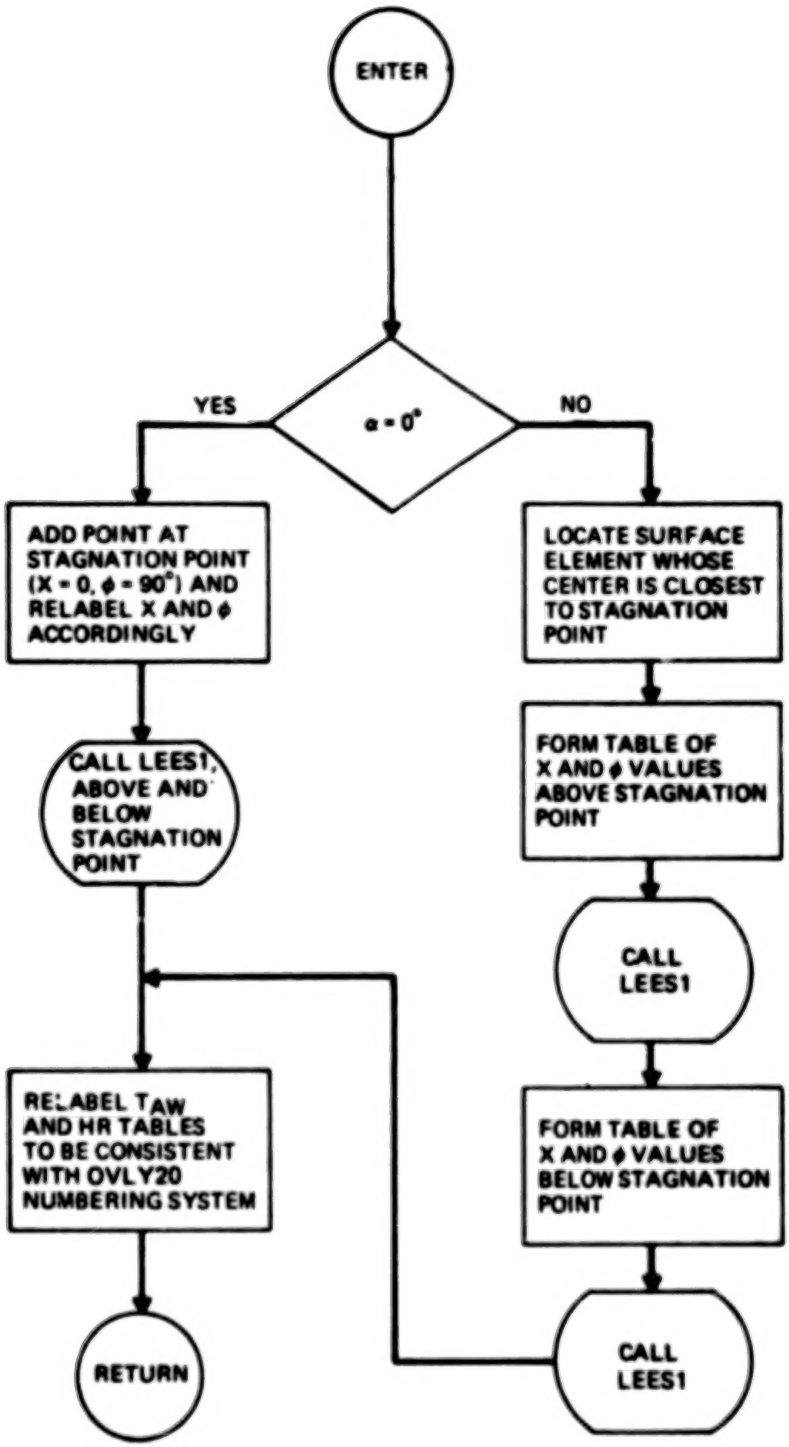

ENTER
α = 0°
YES
NO
ADD POINT AT STAGNATION POINT (X = 0, ϕ = 90°) AND RELABEL X AND ϕ ACCORDINGLY
CALL LEES1, ABOVE AND BELOW STAGNATION POINT
RELABEL TAW AND HR TABLES TO BE CONSISTENT WITH OVLY20 NUMBERING SYSTEM
RETURN
LOCATE SURFACE ELEMENT WHOSE CENTER IS CLOSEST TO STAGNATION POINT
FORM TABLE OF X AND ϕ VALUES ABOVE STAGNATION POINT
CALL LEES1
FORM TABLE OF X AND ϕ VALUES BELOW STAGNATION POINT
CALL LEES1

FIG. D-13 SUBROUTINE ATTAC2 FLOW CHART

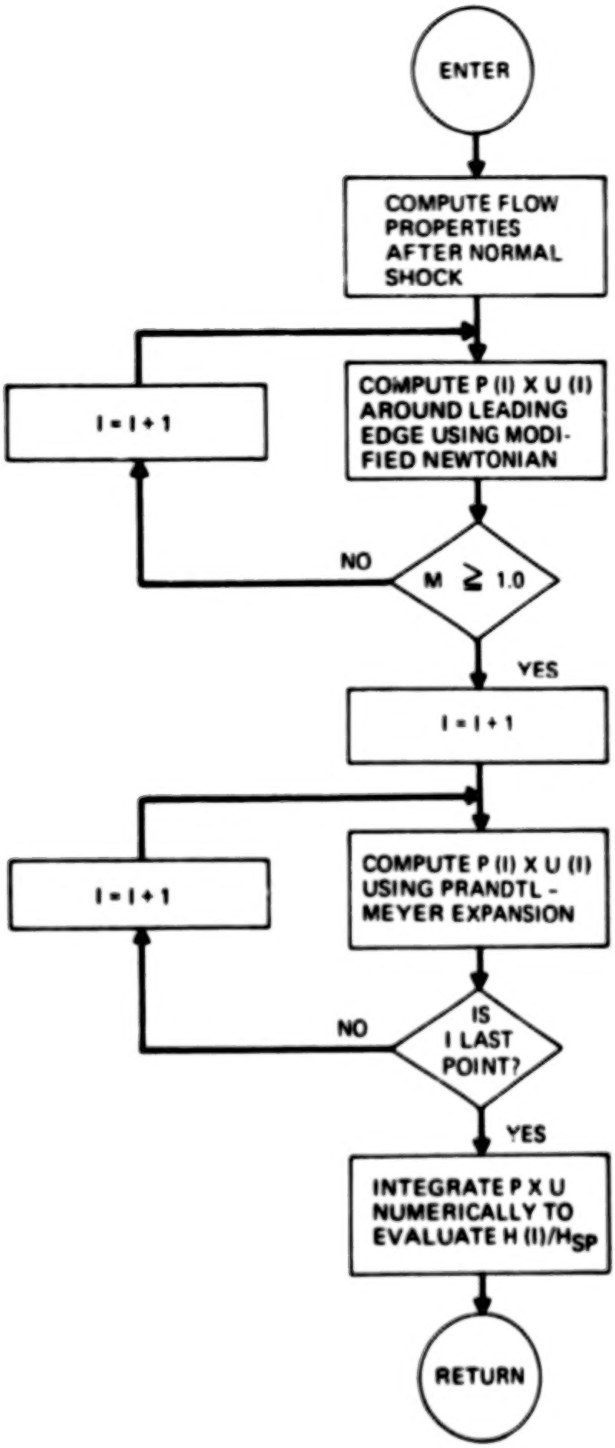

ENTER
COMPUTE FLOW PROPERTIES AFTER NORMAL SHOCK
COMPUTE P (I) X U (I) AROUND LEADING EDGE USING MODI-FIED NEWTONIAN
I = I + 1
NO
M ≥ 1.0
YES
I = I + 1
COMPUTE P (I) X U (I) USING PRANDTL - MEYER EXPANSION
I = I + 1
NO
IS I LAST POINT?
YES
INTEGRATE P X U NUMERICALLY TO EVALUATE H (I)/H_{SP}
RETURN

FIG. D-14 SUBROUTINE LEES1 FLOW CHART

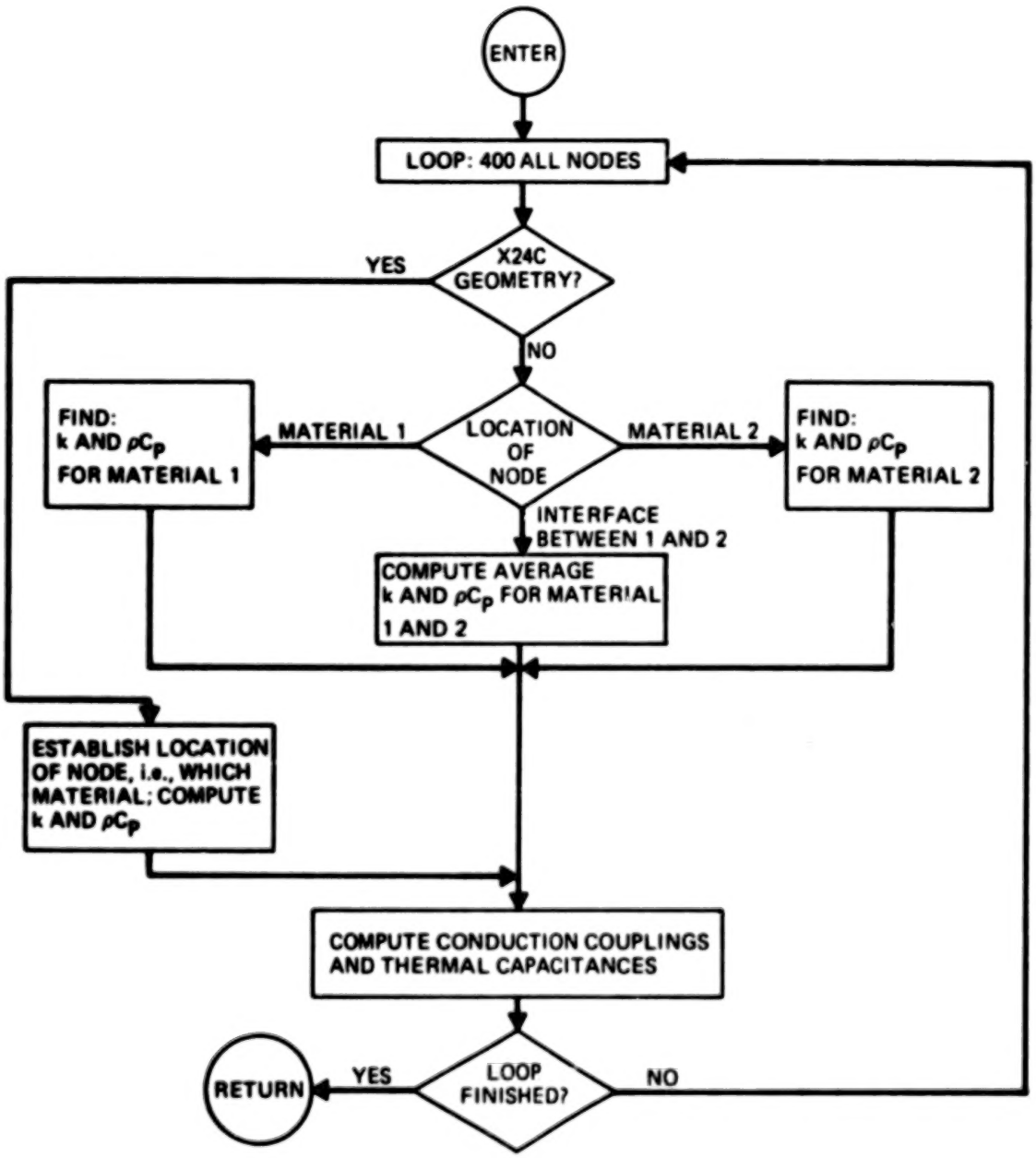


FIG. D-15 SUBROUTINE PROP FLOW CHART

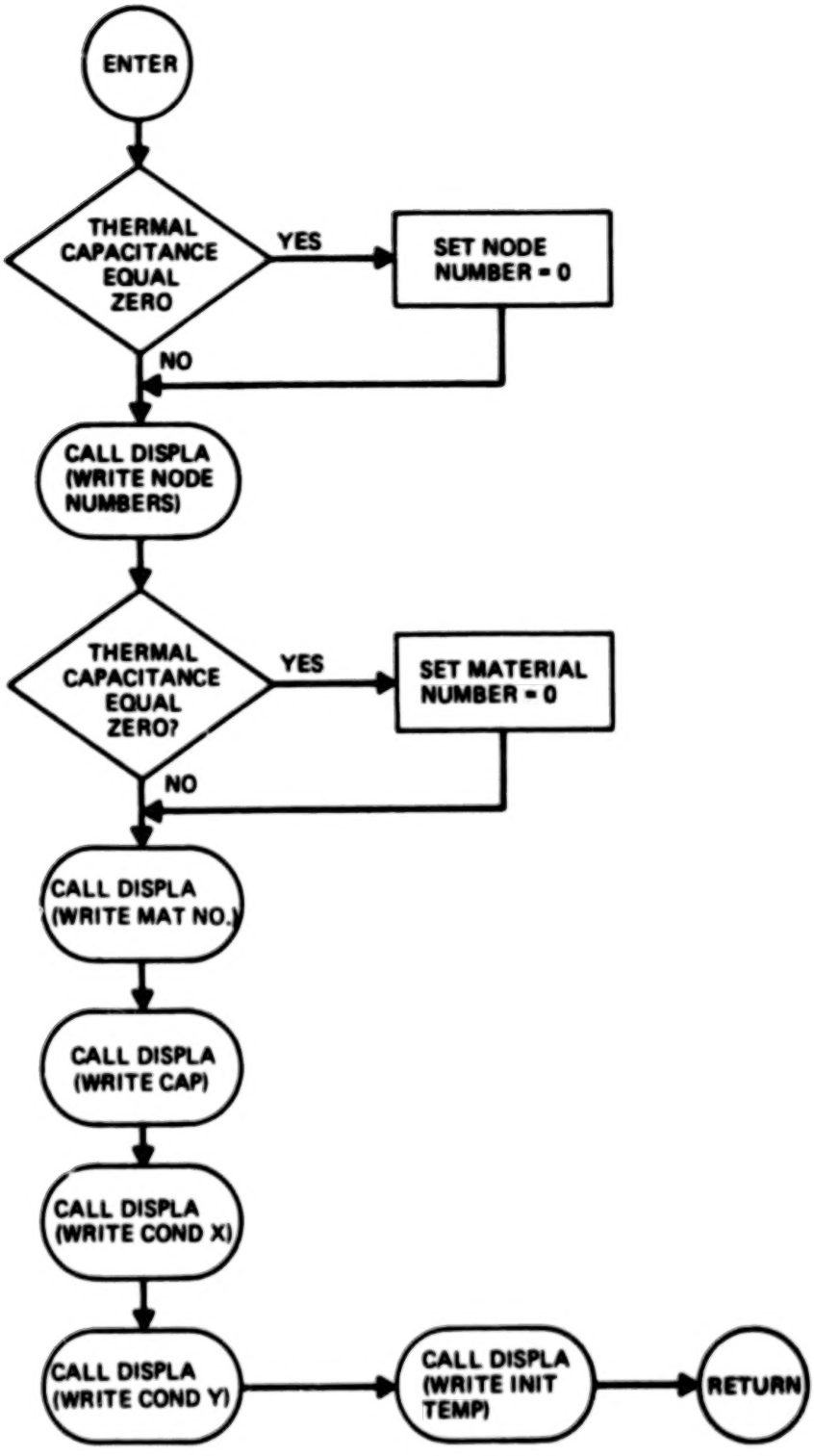


FIG. D-16 SUBROUTINE PRPOUT FLOW CHART

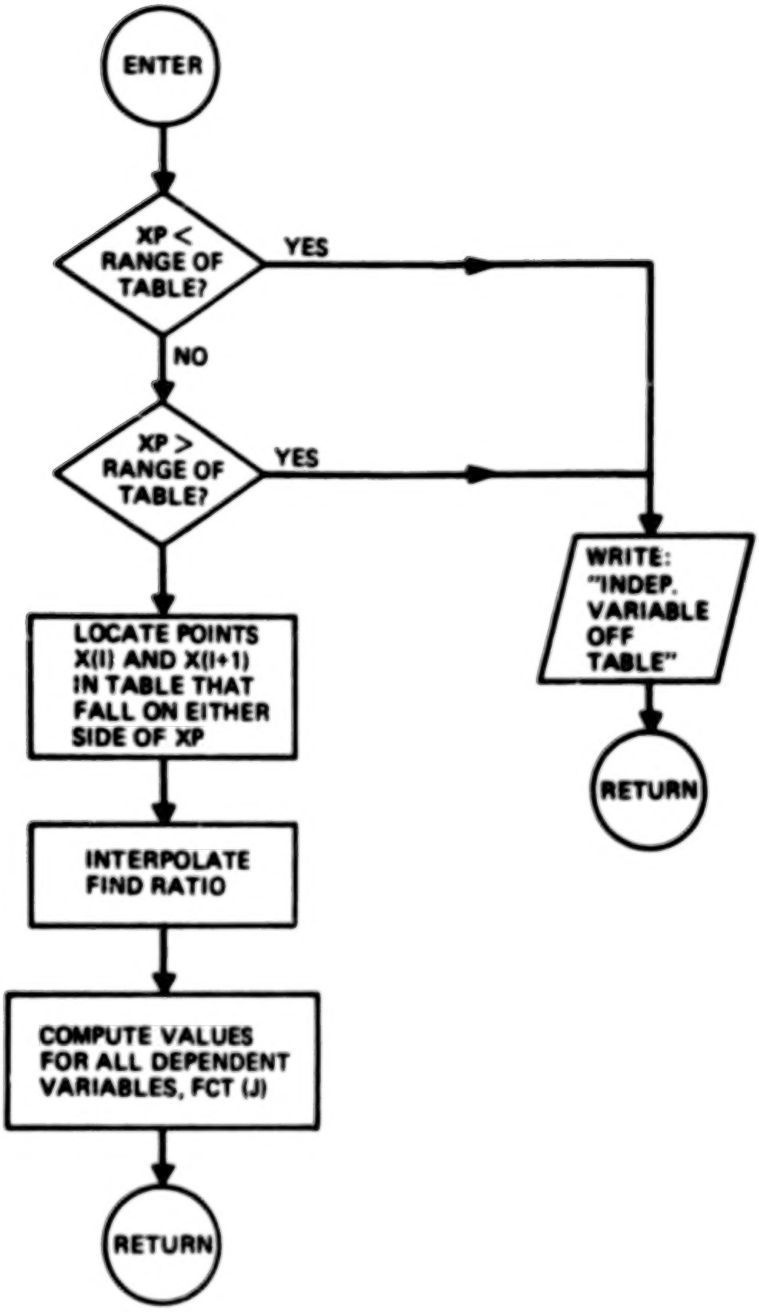


FIG. D-17 SUBROUTINE XINTP1 FLOW CHART

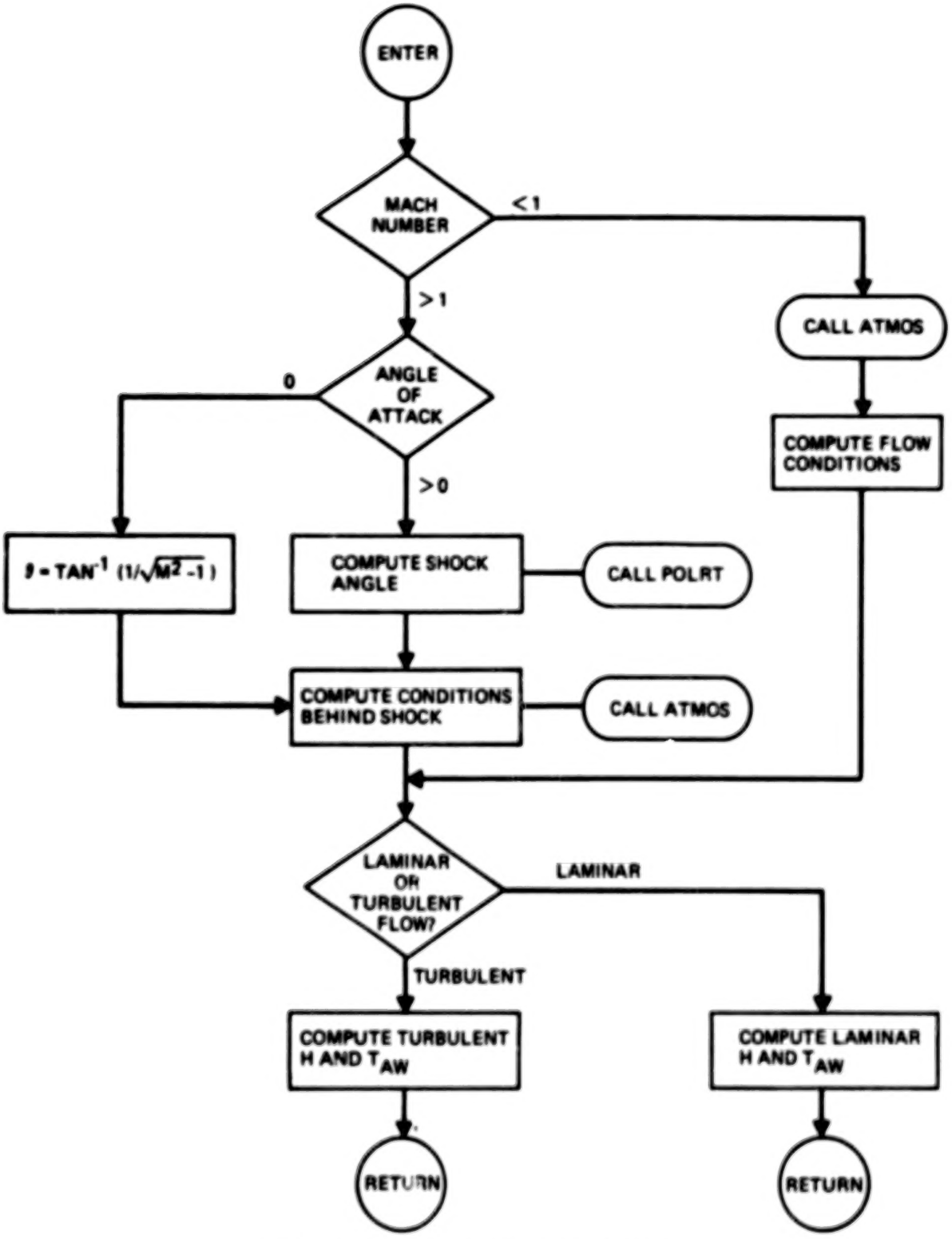


FIG. D-18 SUBROUTINE FLATH FLOW CHART

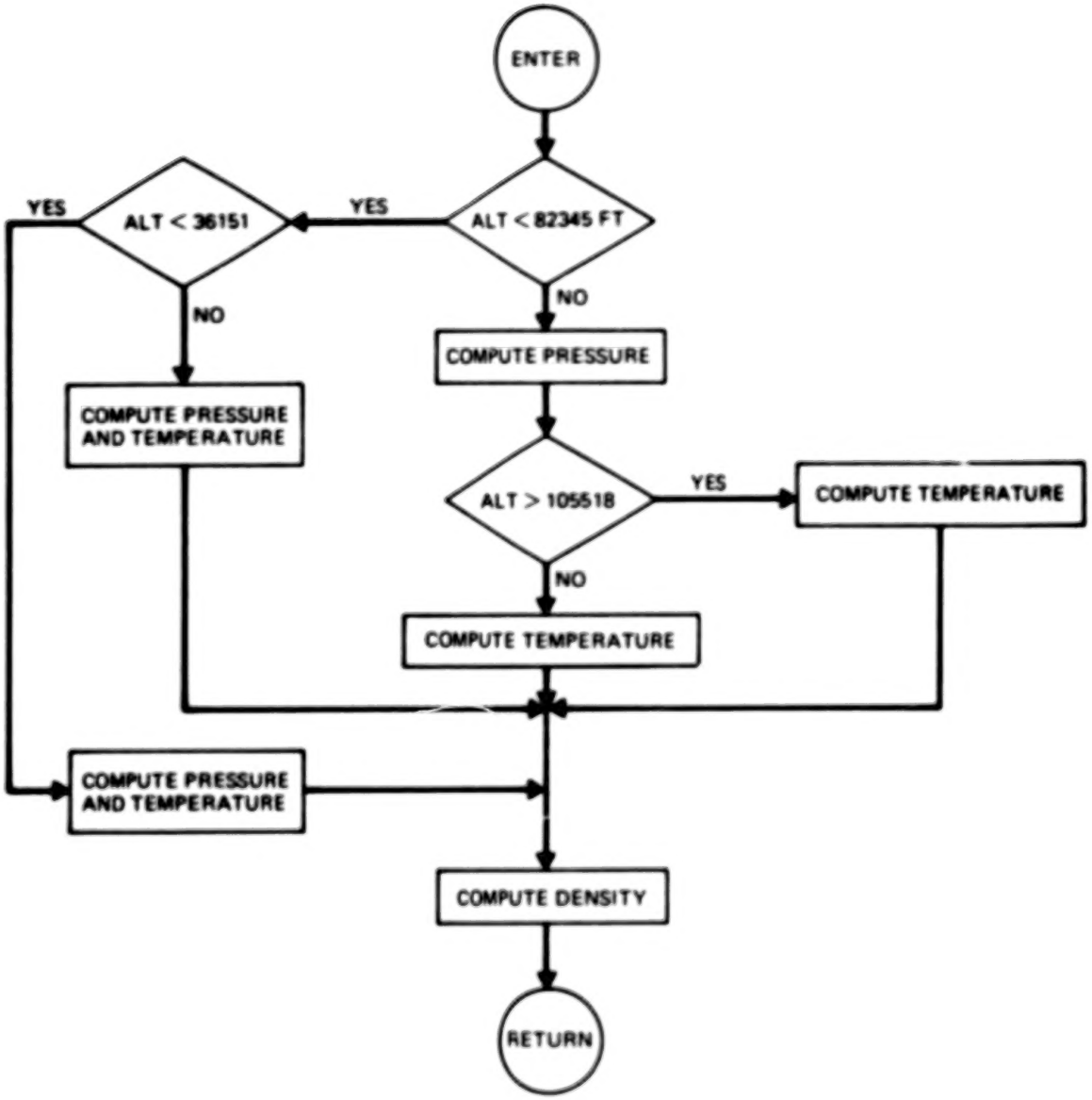


FIG. D-19 SUBROUTINE ATMOS FLOW CHART

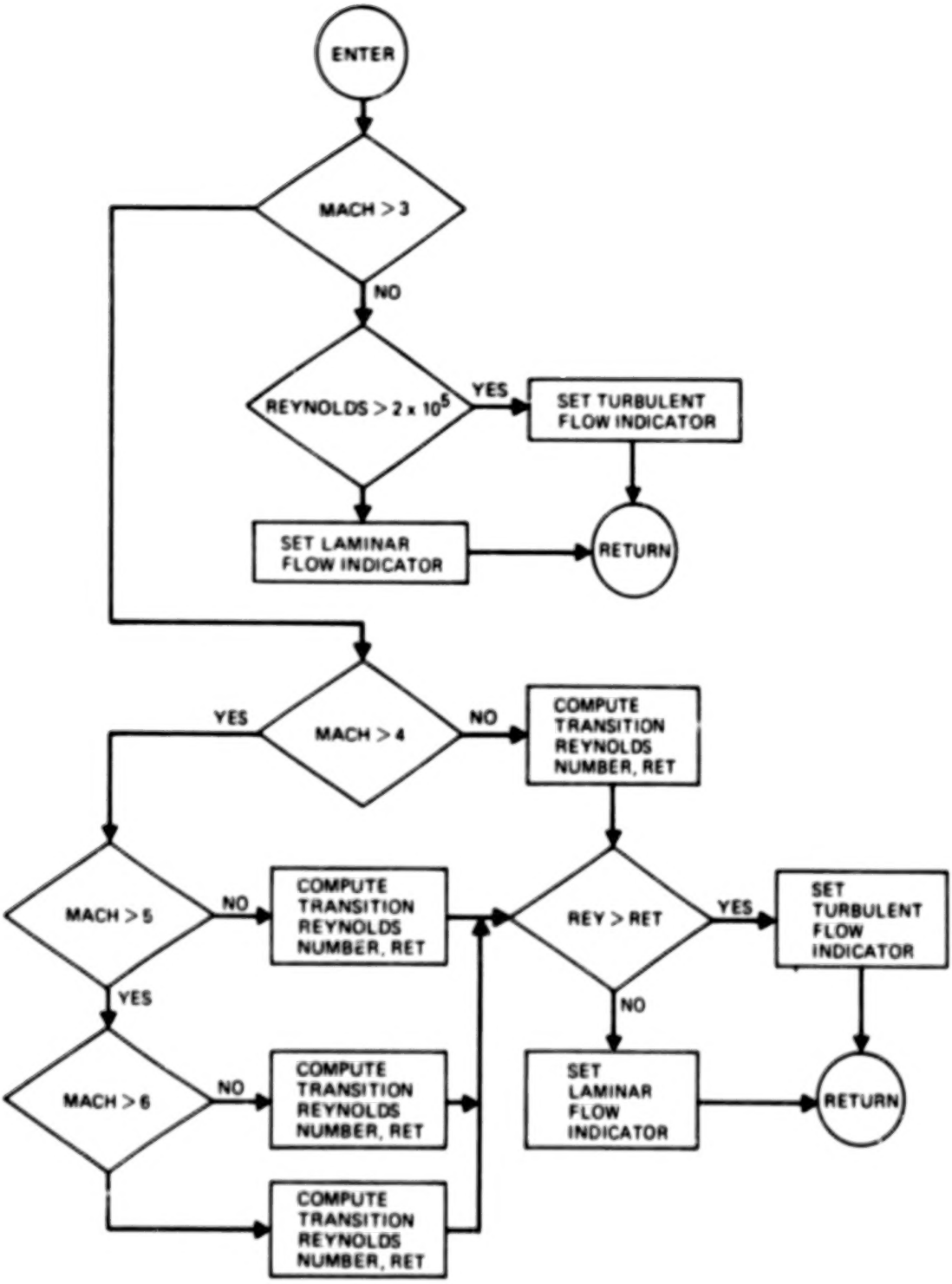


FIG. D-20 SUBROUTINE TRANS FLOW CHART

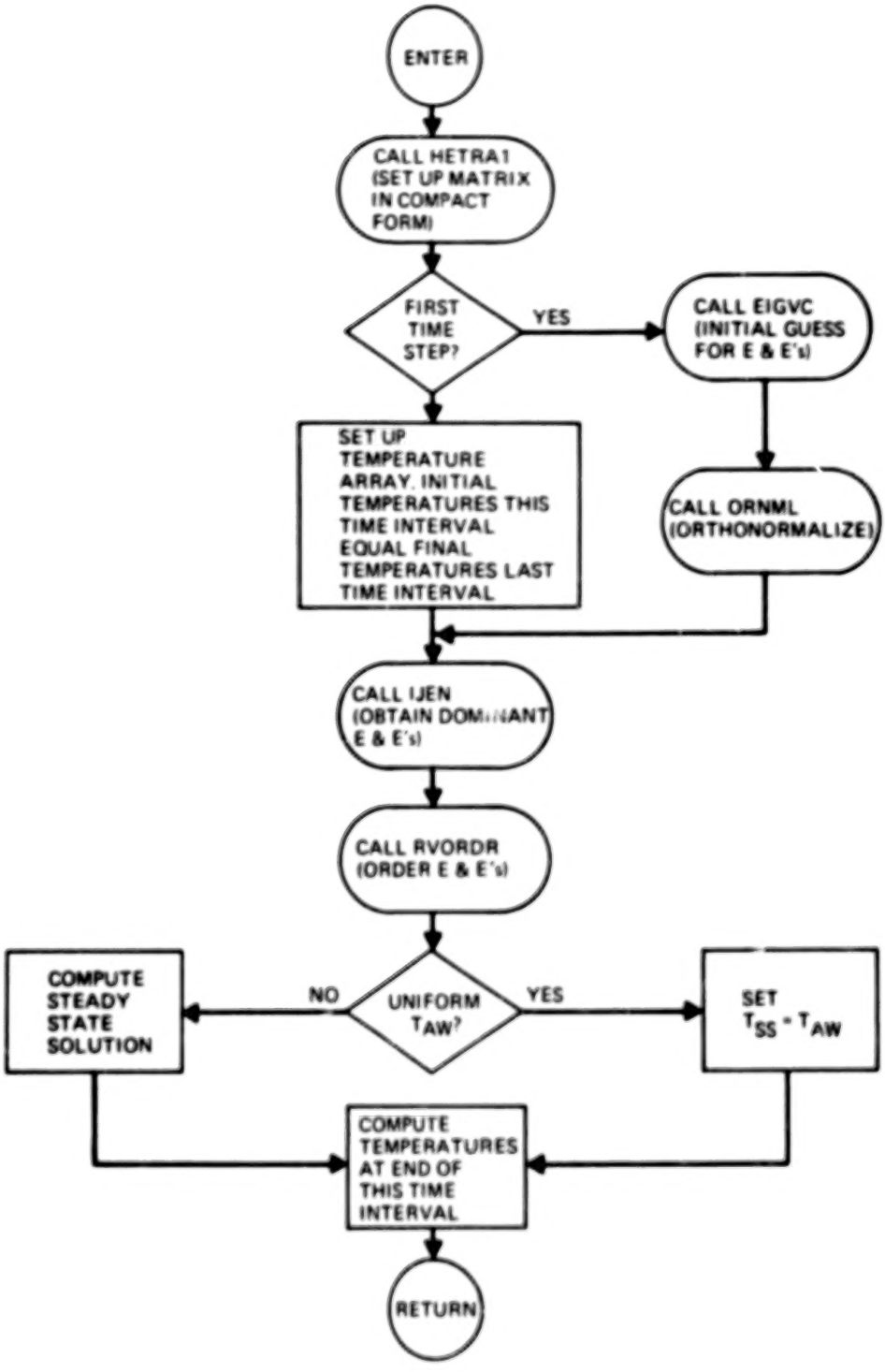

ENTER
CALL HETRA1 (SET UP MATRIX IN COMPACT FORM)
FIRST TIME STEP?
YES
CALL EIGVC (INITIAL GUESS FOR E & E's)
SET UP TEMPERATURE ARRAY. INITIAL TEMPERATURES THIS TIME INTERVAL EQUAL FINAL TEMPERATURES LAST TIME INTERVAL
CALL ORNML (ORTHONORMALIZE)
CALL IJEN (OBTAIN DOMINANT E & E's)
CALL RVORDR (ORDER E & E's)
COMPUTE STEADY STATE SOLUTION
NO
UNIFORM T_{AW}?
YES
SET $T_{SS} = T_{AW}$
COMPUTE TEMPERATURES AT END OF THIS TIME INTERVAL
RETURN

FIG. D-21 SUBROUTINE DESDA1 FLOW CHART

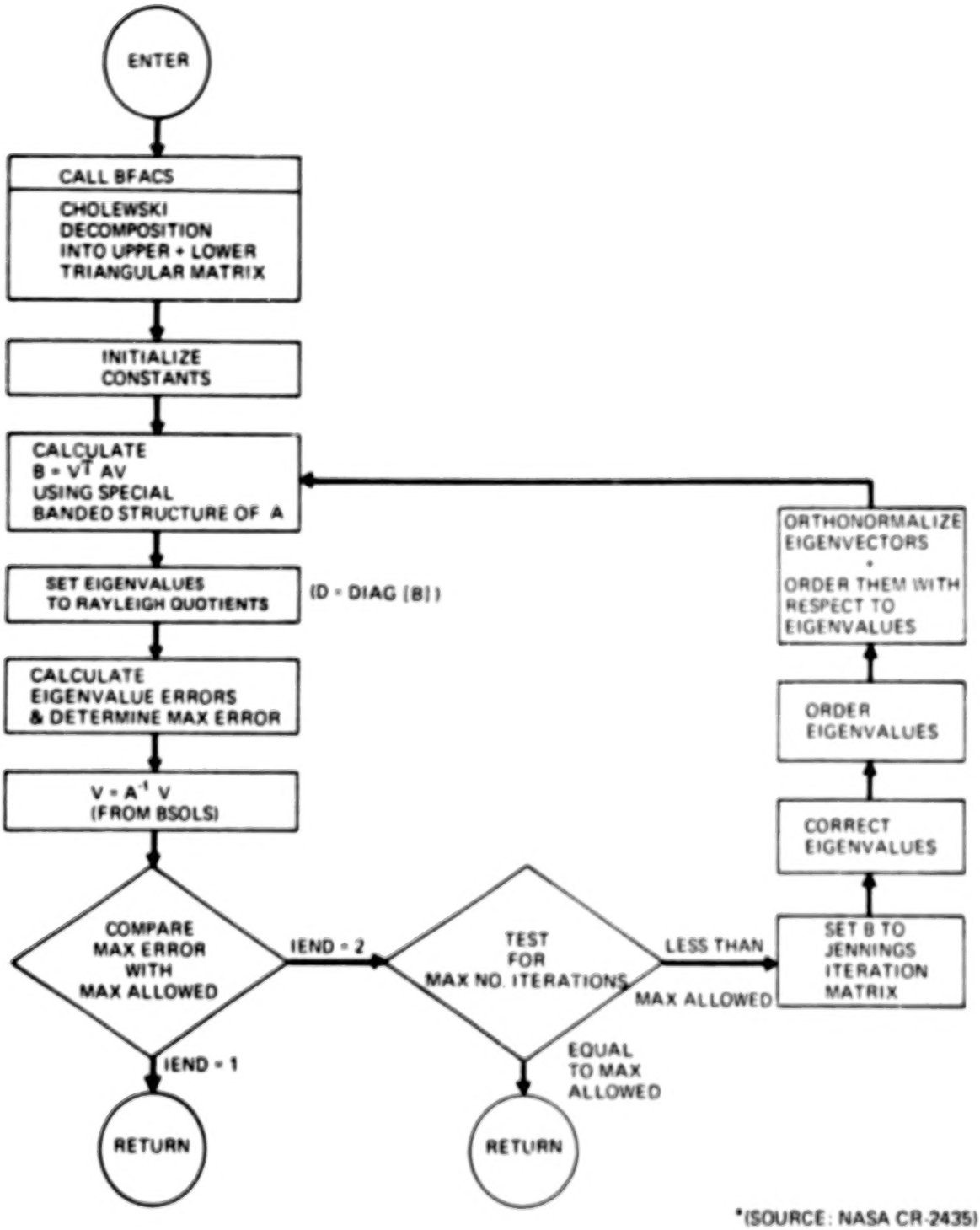
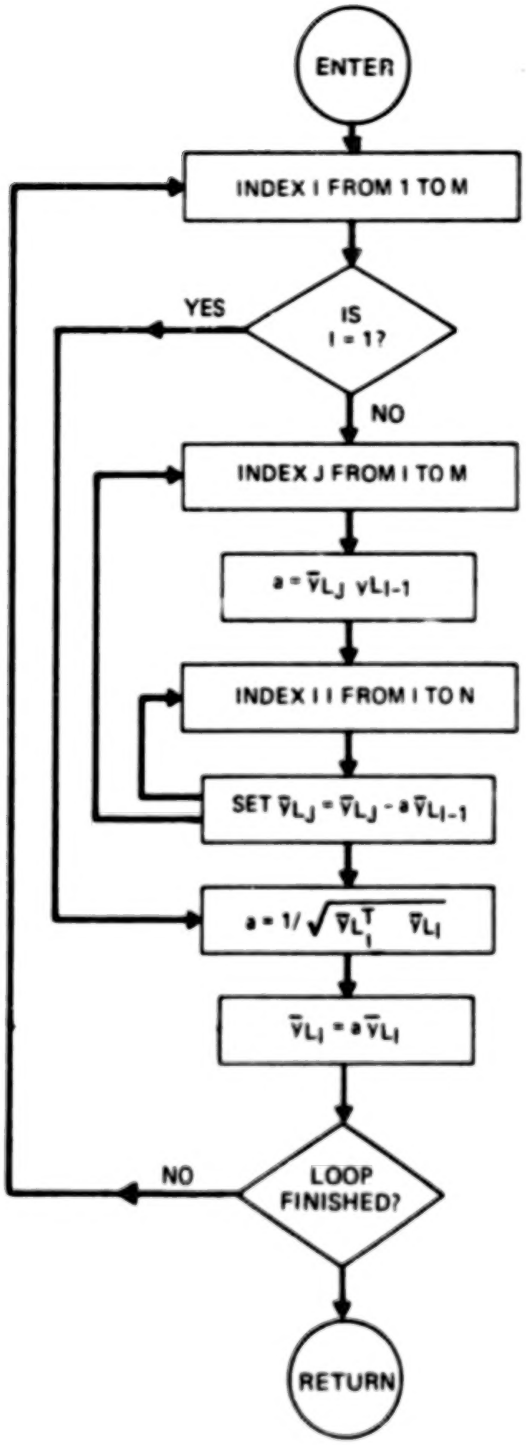

ENTER
CALL BFACS
CHOLEWSKI DECOMPOSITION INTO UPPER + LOWER TRIANGULAR MATRIX
INITIALIZE CONSTANTS
CALCULATE B = V^T AV USING SPECIAL BANDED STRUCTURE OF A
SET EIGENVALUES TO RAYLEIGH QUOTIENTS
(D = DIAG [B])
CALCULATE EIGENVALUE ERRORS & DETERMINE MAX ERROR
V = A^-1 V (FROM BSOLS)
COMPARE MAX ERROR WITH MAX ALLOWED
IEND = 2
IEND = 1
TEST FOR MAX NO. ITERATIONS
LESS THAN MAX ALLOWED
EQUAL TO MAX ALLOWED
SET B TO JENNINGS ITERATION MATRIX
CORRECT EIGENVALUES
ORDER EIGENVALUES
ORTHONORMALIZE EIGENVECTORS + ORDER THEM WITH RESPECT TO EIGENVALUES
RETURN
RETURN
*(SOURCE: NASA CR-2435)

FIG. D-22 SUBROUTINE IJEN FLOW CHART*



*(SOURCE: NASA CR-2435)

FIG. D-23 SUBROUTINE ORNML FLOW CHART*

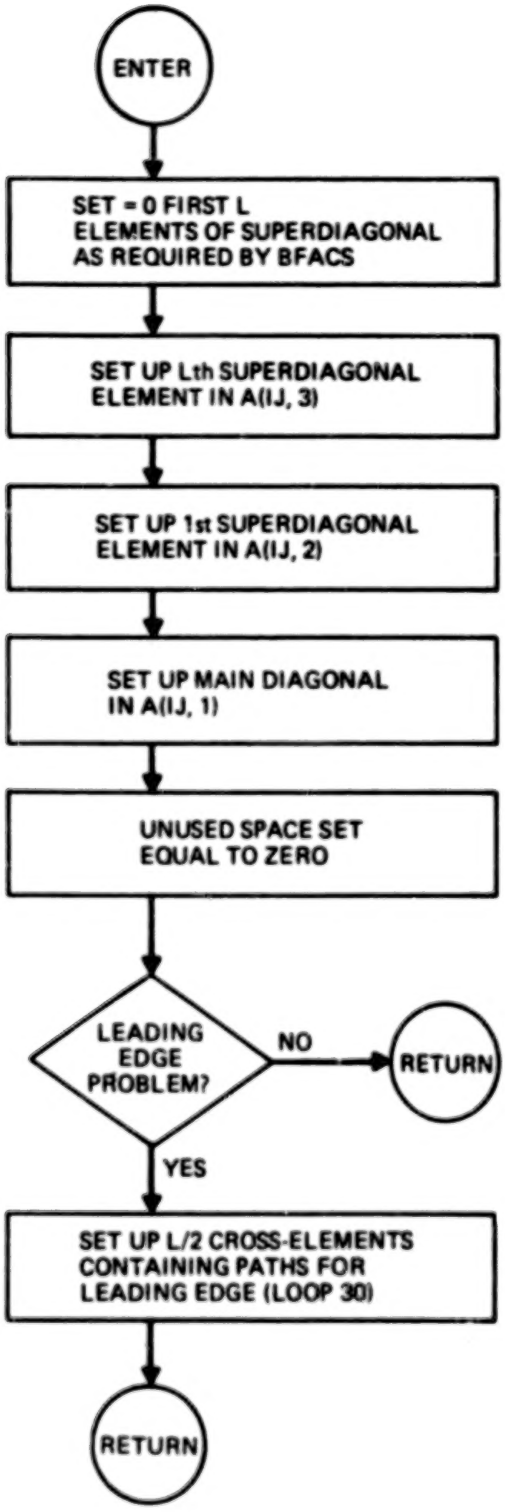
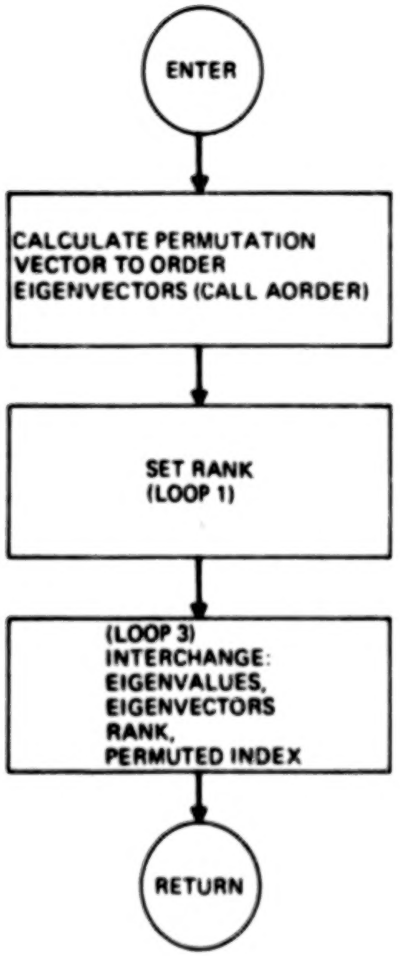

ENTER
SET = 0 FIRST L ELEMENTS OF SUPERDIAGONAL AS REQUIRED BY BFACS
SET UP Lth SUPERDIAGONAL ELEMENT IN A(IJ, 3)
SET UP 1st SUPERDIAGONAL ELEMENT IN A(IJ, 2)
SET UP MAIN DIAGONAL IN A(IJ, 1)
UNUSED SPACE SET EQUAL TO ZERO
LEADING EDGE PROBLEM?
NO
RETURN
YES
SET UP L/2 CROSS-ELEMENTS CONTAINING PATHS FOR LEADING EDGE (LOOP 30)
RETURN

FIG. D-24 SUBROUTINE HETRA1 FLOW CHART

QUANTITY	SYMBOL	INPUT/OUTPUT	DIMENSION
EIGENVALUES	R	IN + OUT	R(MM)
EIGENVECTORS	V	IN + OUT	V(MID, MM)
RANK VECTOR	K	OUT	K(MM)
PERMUTATION VECTOR	L	OUT	L(MM)
EIGENVECTOR DIMENSION	N	IN	–
NUMBER OF EIGENVECTORS	MM	IN	
DIMENSION OF ARRAY USED TO STORE EIGENVECTORS	MID	IN	


ENTER
CALCULATE PERMUTATION VECTOR TO ORDER EIGENVECTORS (CALL AORDER)
SET RANK (LOOP 1)
(LOOP 3) INTERCHANGE: EIGENVALUES, EIGENVECTORS RANK, PERMUTED INDEX
RETURN

*(SOURCE: NASA CR-2435)

FIG. D-25 SUBROUTINE R'ORDR FLOW CHART*

SCAPR2 CALCULATES THE INNER
PRODUCT OF TWO VECTORS STORED
AS EQUALLY SPACED WORDS IN FORTRAN ARRAYS

*(SOURCE: NASA CR-2435)

FIG. D-26 SUBROUTINE SCAPR2 FLOW CHART*

Sheet D-1. Subroutine NURED Description (Sheet 1 of 4)

Purpose

To read a set of tables for functions 2 variables

Program Description

The input data must be structured as specified below. The calling program must contain the statement, COMMON STG(L), where $L \geq (L1(1)+1)\ (L2(1)+1) + \ldots + (L1(M)+1)\ (L2(M)+1)$. Program restrictions are noted below:

Input Parameters

FORTRAN Name	Description
NUMTBL	= 1 for first call to NURED = K for replacement of tables, where K is the table number of the first table being replaced
MANDAN	= 0 for initial read in = 1 for table replacement
L1	Array of dimension M, where M is the maximum number of tables in storage at any one time L1(K) = number of x_i in table K
L2	Array of dimension M L2(K) = number of y_i in table K
NUMPTS	Array of dimension M+1 NUMPTS(K) = the number of table entries preceding table K

Output Parameters

NG	To be set to 0 in the calling program NG = 0 if tables have been read correctly NG = 1 if there has been a read in error

Calling Sequence

CALL NURED (NUMTBL, MANDAN, NG, L1, L2, NUMPTS)

Input Format

For each table the functions values, F, (X,Y) are entered for consecutive groups of nine values of X, over all values of Y

(continued)

Sheet D-1. Subroutine NURED Description (Sheet 2 of 4)

Input Format (Cont'd)

The input data must be structured as indicated. First, there is a header card with the appropriate entries in the indicated columns

1–2	3–4	17 70	71–72
L1	L2	COMMENTS	Seq #

Next, for the first 9 values of X the input data takes the form

1–7	8–14	15–21	22–28	...	57–63	64–70	71–72
	X_1	X_2	X_3	...	X_8	X_9	Seq #
Y_1	F(1,1)	F(2,1)	F(3,1)	...	F(8,1)	F(9,1)	Seq #
.	.	.	.	...	.	.	.
.	.	.	.	...	.	.	.
Y_{L2}	F(1,L2)	F(2,L2)	F(3,L2)	...	F(8,L2)	F(9,L2)	Seq #

For the second group of 9 values of X, the form of the input is

	X_{10}	X_{11}	X_{12}	...	X_{17}	X_{18}	Seq #
Y_1	F(10,1)	F(11,1)	F(12,1)	...	F(17,1)	F(18,1)	Seq #
.	.	.	.	...	.	.	.
.	.	.	.	...	.	.	.
Y_{L2}	F(10,L2)	F(11,L2)	F(12,L2)	...	F(17,L2)	F(18,L2)	Seq #

Additional values of X are handled similarly. When all X values have been accounted for, the next table of values for the next function is set up in the same way. When all data tables which are to be read in at any one time have been set up as indicated, a blank card is placed at the end of the data deck.

The parameters used above are defined as follows:

L1	= Number of X values in table	(I2)
L2	= Number of Y values in table	(I2)
Seq #	= Sequence number of a card within a table, beginning with 0 for the first card	(I2)
F(I,J)	= Function value for X_I, Y_J	(E7.0)
X_I	= Argument 1 values in table	(E7.0)
Y_I	= Argument 2 values in table	(E7.0)

The figure illustrates the card format for the tables (see Sheet 4 of Sheet D-1).

(continued)

Sheet D-1. Subroutine NURED Description (Sheet 3 of 4)

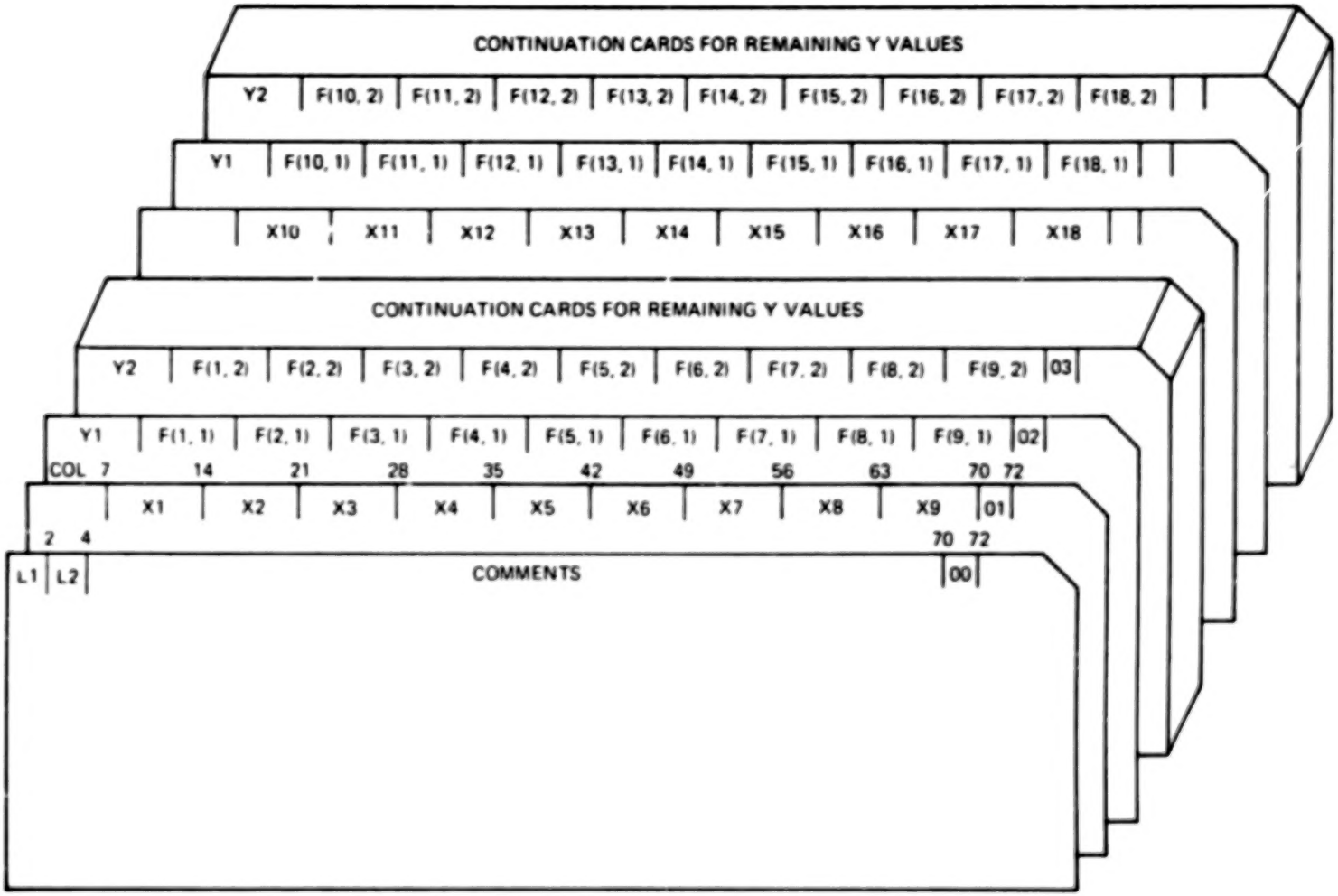
Program Restrictions

The tabular values of the variables X and Y must appear in algebraically increasing order. The variables X and Y, and the function values must:

- Be single precision numbers less than 99999E9
- Have a maximum of 7 significant digits if positive
- Have a maximum of 6 significant digits if negative

A maximum of 99 cards is allowed for each table

CARD FORMAT FOR TABLES



SHEET D-1 SUBROUTINE NURED DESCRIPTION (SHEET 4 of 4)

Sheet D-2. Subroutine DINTK Description (Sheet 1 of 3)

Purpose

Table lookup and linear interpolation for several functions of two variables.

Analytic Description

In the derivation of STINT (6.1.1.5) equations, we find the linear interpolation form

$$f(x,y) = \frac{f(x,1)\,(y - y_0) + f(x,0)\,(y_1 - y)}{y_1 - y_0}$$ which, by algebraic manipulation becomes

$$\text{(i)} \quad f(x,y) = \frac{(y_1 - y)\,(y_0 - y)}{y_1 - y_0} \left[\frac{f(x,0)}{y_0 - y} - \frac{f(x,1)}{y_1 - y}\right]$$

Analytic Restrictions

The function f should be linear

Program Description

$f_1(x,y)$, $f_2(x,y)$, ..., $f_m(x,y)$ are found for $x_i \le x \le x_{i+1}$ and $y_i \le y \le y_{i+1}$, using equation (i)

Program Restrictions

Extrapolation will not be performed. The calling program must contain the statement COMMON STG(L), where

$$L \ge \sum_{i=1}^{m} (L1(i) + 1)\,(L2(i) + 1)$$

Input Parameters

FORTRAN Name	Description
L1, L2, NUMPTS	As described in NURED
KODE	Dummy array of length m. Initialize to zero in the calling program
N1HIB4	Dummy array of length m
N2HIB4	Dummy array of length m
ARG1	Value of x argument
ARG2	Value of y argument
NUMTBL	Number of the first table in which interpolation will take place
L3	Number of functions to be interpolated

(continued)

Sheet D-2. Subroutine DINTK Description (Sheet 2 of 3)

Output Parameters

FCT	Array of length L3, consisting of the elements FCT(1) = $F_j(x,y)$, ..., FCT(L3) = $f_n(x,y)$. (j=NUMTBL, n=j+L3-1)
NG	To be initialized to zero before calling DINTK NG = 0 indicates a normal return NG = 2 indicates a machine error or an error in the calling sequence NG = 3 indicates that x or y is outside of the range of the tables

Library Supplied Routines

User must call NURED to read in the tables of x, y, $f_1(x,y)$, ..., $f_m(x,y)$ prior to calling DINTK

Calling Sequence

CALL DINTK (L1, L2, NUMPTS, KODE, N1HIB4, N2HIB4, ARG1, ARG2, NUMTBL, L3, FCT, NG)

List of Variables

FORTRAN Name	Description
L1	
L2	
NUMPTS	
KODE	
N1HIB4	
N2HIB4	
ARG1	
ARG2	
NUMTBL	
L3	
FCT	
NG	
STG	Table of values of input
X	Temporary storage for ARG1
Y	Temporary storage for ARG2
ANS	Working storage
FACTOR	Working storage
F	Working storage
NT	
NCHECK1	
NCHECK2	
INC	Indices
K	
NLO1	
NLO2	
NBLO1	
NBLO2	

(continued)

Sheet D-2. Subroutine DINTK Description (Cont'd) (Sheet 3 of 3)

List of Variables (Cont'd)

FORTRAN Name	Description
NHI1	Indices
NHI2	
JINDEX	
KINDEX	
I	
J	
M	
LDUMY1	
LDUMY2	
LENGTH	
LIMLOW	
LIMUPR	
KTEST1	DO loop indices
KTEST2	
KTEST3	
KTEST4	
L	
M	
I	
J	
LA	Working storage
LB	
NDUMY1	
NDUMY2	

Sheet D-3. Subroutine POLRT Description

Purpose

Computes the real and complex roots of a real polynominal

Usage

Call POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)

Description of Parameters

XCOF = Vector of M+1 coefficients of the polynominal ordered from smallest to largest power
COF = Working vector of length M+1
M = Order of polynominal
ROOTR = Resultant vector of length M containing real roots of the polynominal
ROOTI = Resultant vector of length M containing the corresponding imaginary roots of the polynominal
IER = Error code where:

IER=0	No error
IER=1	M less than one
IER=2	M greater than 36
IER=3	Unable to determine root with 500 interations on 5 starting values
IER=4	High order coefficient is zero

Remarks

Limited to 36th order polynominal or less
Floating point overflow may occur for high order polynominals but will not affect the accuracy of the results

Subroutines and Function Subprograms Required

None

Method

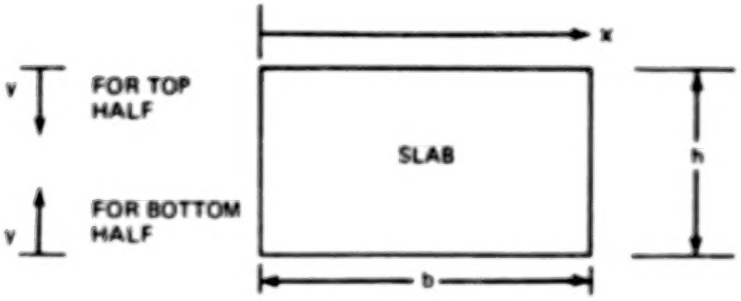
Newton-Raphson iterative technique. The final iterations on each root are performed using the original polynominal rather than the reduced polynominal to avoid accumulated errors in the reduced polynominal

Sheet D-4. Subroutine EIGVC Description (From NASA CR-2435)

EIGVC computes guesses of the eigenvalues, eigenvectors and associated permutation index that are necessary to start the iteration in the Jennings method to calculate eigenvalues and eigenvectors. The formulae used for these guesses are:

ith eigenvalue $R \simeq -i$

ith eigenvector $A \simeq e^{+y/(h/2)} \sin\left\{\frac{\pi}{2} + \pi\frac{x}{b}(i-1)\right\}$



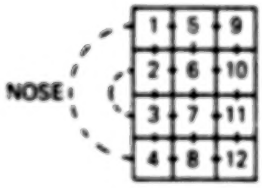
Sheet D-5. Subroutine BFACS Description (Sheet 1 of 2) (From NASA CR-2435)

Given a special, L-banded, positive or negative definite symmetric N-th order matrix, S, decompose it into the product:

$$S = U^T D^{-1} U$$

where U is an L-banded nonsingular upper triangular matrix with unit diagonal elements, and D is a nonsingular diagonal matrix.

The S matrix is inputted as elements of three one-dimensional arrays, A, B, E. The N elements of A are the main diagonal elements of S, the leading (N−L) elements of E are the L-th super- (and sub-) diagonal elements of S. The N-1 elements of B are super- (and sub-) diagonal elements of S. *The trailing L/2 elements of E (optionally) define main cross diagonal elements of the upper L-th order submatrix of S. In general, later definitions override earlier one, e.g. if L = 1, B (not E) defines the super diagonal elements of S. In the case for which BFACS is intended, most elements inside the band are zero. The cross diagonal is installed only if the argument, α, is not zero. The S matrix is topologically equivalent to conduction paths in a slab (leading edge if $\alpha \neq 0$); consider the N = 12, L = 4 example:



Note: $b_4 = b_8 = 0$ for the conduction problem generally but must be explicitly made 0 for BFACS

Note: If nose paths are included e_{11}, e_{12} are used ($\alpha \neq 0$).

	1	2	3	4	5	6	7	8	9	10	11	12
1	a_1	b_1	0	e_{12}	e_1	0	0	0	0	0	0	0
2	b_1	a_2	e_{11}	0	0	e_2	0	0	0	0	0	0
3	0	e_{11}	a_3	b_3	0	0	e_3	0	0	0	0	0
4	e_{12}	0	b_3	a_4	b_4	0	0	e_4	0	0	0	0
5	e_1	0	0	b_4	a_5	b_5	0	0	e_5	0	0	0
6	0	e_2	0	0	b_5	a_6	b_6	0	0	e_6	0	0
7	0	0	e_3	0	0	b_6	a_7	b_7	0	0	e_7	0
8	0	0	0	e_4	0	0	b_7	a_8	b_8	0	0	e_8
9	0	0	0	0	e_5	0	0	b_8	a_9	b_9	0	0
10	0	0	0	0	0	e_6	0	0	b_9	a_{10}	b_{10}	0
11	0	0	0	0	0	0	e_7	0	0	b_{10}	a_{11}	b_{11}
12	0	0	0	0	0	0	0	e_8	0	0.	b_{11}	a_{12}

Sheet D-5. Subroutine BFACS Description (Sheet 2 of 2) (From NASA CR-2435)

To take advantage of both symmetry and the band form U is stored in a rectangular array of size MID by L, where MID $\geq$ N, the bottom row is used as scratch storage, $r_1, r_2, \ldots, r_L$, and the unused bottom triangle is zeroed out (for convenience in printing only). U appears as

STORED ARRAY

	1	2	3	4
1	u_{11}	u_{12}	u_{13}	u_{14}
2	u_{21}	u_{22}	u_{23}	u_{24}
3	u_{31}	u_{32}	u_{33}	u_{34}
4	u_{41}	u_{42}	u_{43}	u_{44}
5	u_{51}	u_{52}	u_{53}	u_{54}
6	u_{61}	u_{62}	u_{63}	u_{64}
7	u_{71}	u_{72}	u_{73}	u_{74}
8	u_{81}	u_{82}	u_{83}	u_{84}
9	u_{91}	u_{92}	u_{93}	0
10	$u_{10,1}$	$u_{10,2}$	0	0
11	$u_{11,1}$	0	0	0
12	r_1	r_2	r_3	r_4

NON ZERO ELEMENTS OF U MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	u_{11}	u_{12}	u_{13}	u_{14}							
2		1	u_{21}	u_{22}	u_{23}	u_{24}						
3			1	u_{31}	u_{32}	u_{33}	u_{34}					
4				1	u_{41}	u_{42}	u_{43}	u_{44}				
5					1	u_{51}	u_{52}	u_{53}	u_{54}			
6						1	u_{61}	u_{62}	u_{63}	u_{64}		
7							1	u_{71}	u_{72}	u_{73}	u_{74}	
8								1	u_{81}	u_{82}	u_{83}	u_{84}
9									1	u_{91}	u_{92}	u_{93}
10										1	$u_{10,1}$	$u_{10,2}$
11											1	$u_{11,1}$
12												1

The N elements of D^{-1} are stored in an N-array. For the usual case of S being either positive-or negative-definite, these elements are all positive or all negative, respectively. However the routine will "work" provided only that the leading N principle minors are non-zero. For details see the following article which guarantees high accuracy only for the definite cases of usual interest: "Symmetric Decomposition of Positive Definite Band Matrices", R.S. Martin, J.H. Wilkinson, C. I/4, LINEAR ALGEBRA – HANDBOOK FOR AUTOMATIC COMPUTATION, VOLUME II, Springer-Verlag, 1971.

Sheet D-6. Subroutine BSOLS Description (From NASA CR-2435)

Given the product form decomposition of an L-banded symmetric matrix, $S = U^T D^{-1} U$, as calculated by the BFACS subroutine, BSOLS solves a system of N linear equations with M right hand sides:
$S \{Y_1\ Y_2 \cdots YM\} = \{Y_1\ Y_2 \cdots YM\}$

The routine simply carries out the standard forward substitution phase:

$$z = U^{-T} y$$

followed by the standard backward substitution phase:

$$x = U^{-1} D z$$

The only unusual aspect is the rather unorthodox storage scheme which is described in the documentation for subroutine BFACS. This scheme is necessary to exploit the banded symmetric form of S in the most efficient way in terms of computer memory. For details see: R.S. Martin, J.H. Wilkinson, "Symmetric Decomposition of Positive Definite Bank Matrices", in: Linear Algebra–Handbook for Automatic Computation, Volume II, C. I/4, Springer-Verlug, 1971

Sheet D-7. Subroutine AORDER Description (From NASA CR-2435)

PURPOSE: ORDER A SET OF REAL NUMBERS

CALLING SEQUENCE: CALL AORDER (A, N, IPERM)

	NAME	DIMENSION	DESCRIPTION
INPUT	A	A(N)	ELEMENTS TO BE ORDERED
	N		NUMBER OF ELEMENTS = | N | N > 0 INCREASING ORDER N < 0 DECREASING ORDER
OUTPUT	IPERM	IPERM(N)	ORDER VECTOR - SPECIFIES THE SEQUENCE OF ELEMENT INDEX NUMBERS WHICH WILL PRESENT A AS AN ORDERED SET, i.e. DO 100 I = 1, N 100 WRITE (6,1) A (IPERM(I)) 1 FORMAT (F 10.5) WILL LIST A AS AN ORDERED ARRAY

AORDER CALLS NO OTHER SUBROUTINES

Sheet D-8. Subroutine DISPLA Description (From NASA CR-2435)

TITLE: DISPLA – Prints scalars, vectors, rectangular matrices, packed symmetric matrices, and Hessenberg matrices.

AUTHOR: M. J. Rossi

DATE: September 1973

APPLICABLE COMPUTERS: IBM 360/370; CDC 6000 SERIES

SOURCE LANGUAGE: FORTRAN IV

PURPOSE: To simplify printing of mathematical types of data structures in an easily read format which allows titles and index labels.

METHOD: FORTRAN looping and write statements which indexes and addresses arrays according to their type.

USAGE: Call DISPLA (X, NFILE, TITLE, KAR, KIND, NROWS, NCOLS, MID).

X – Input – Array of one or more values to be printed
NFILE – Input – FORTRAN unit for printing.
TITLE – Input – Vector of KAR characters used as title.
KAR – Input – Number of characters in above string.
KIND – Input – Type of mathematical data structure:
= 0 scalar (or vector printed on one line with no index)
= 1 vector of |NROWS| elements, indexed
= 2 Rectangular |NROWS| by NCOLS matrix – Dimension (MID, *)
= 3 Packed Symmetric matrix of order |NROWS|

– $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ – lower triangular partial rows if NROWS positive

– $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ – lower triangular partial columns if NROWS negative

= 4 – Transposed Hessenberg matrix of order NROWS – Dimension (MID, MID)

NROWS – Input – Number of elements if KIND = 0 or 1
– Number of rows if KIND = 2
– Matrix order if KIND = 3 or 4
NCOLS – Input – Number of columns if KIND = 2
– Ignored otherwise
MID – Input – Matrix Dimension if KIND = 2 or 4
– Ignored otherwise

SUBROUTINE REQUIRED: SWITCH

Sheet D-9. Subroutine PART Description (From NASA CR-2435)

TITLE: PART -- Prints standard 120 character labels at the top of the next page.

AUTHOR: M. J. Rossi

DATE: September 1973

APPLICABLE COMPUTERS: IBM 360/370; CDC 6000 SERIES

SOURCE LANGUAGE: FORTRAN IV with 2 Assembler Language Subordinate Subroutines.

PURPOSE: To make it convenient to produce standard printed labels with "part" numbers, date, and running CPU time on the top of the next page. Also, prints short line on next line with just CPU time for intermediate timing.

METHOD: On the first printing entry for a given computer run the Date Subroutine is invoked and an 8-character field of an internal word is stored with the date in the form: "KK/LL/MM", where KK is the index number for the month, LL is the day of the month, and MM is the last 2 digits of the year, e.g., March 15, 1973 3/15/73. Also, at this time, the SECOND subroutine is invoked to both establish the zero time point and to set the units to hundreds of a second. Then the first printed page heading is given with a zero time and a PART number 1 reported. Subsequent printing calls will give the time as: NN.II.JJ where NN is the number of minutes elapsed, II is the number of seconds, and JJ is the number of hundredths of seconds. The PART number is incremented by one for each printing call. There are two fields of alphameric information for the full printing mode which are under control of the user: (1) The first is a 40 character LABEL field which is set upon calling PART in the non-printing mode, (2) The second is a 48 character field which is supplied on a full printing call. There is also a partial printing mode which simply results in the appearance on the next line of an 8 character field of user supplied TITLE along with running CPU time.

USAGE: Call PART ('XX. . .X', L)
'XX. . .X' – Input = Alphameric string of either 8, 40, or 48 characters depending on the value of L.
L – Input – FORTRAN unit for printing, if positive
– If zero, simply sets 40 character LABEL field and returns
– If negative, prints 8 character TITLE – 'XX. . .X' – and CPU time on next line and increments PART number.
– If positive, prints DATE, TIME, 40 character LABEL, 48 character TITLE, Part Number and spacers with standard notation.

SUBROUTINES REQUIRED: DATE, SECOND

Sheet D-10 Subroutine Switch Description (From NASA CR-2435)

PROGRAM TITLE: Utility routine for re-arrangement of certain triangular arrays
SUBROUTINE NAME: SWITCH INDEX: 12.6.0.1
ANALYST: F. Nolan
PROGRAMMER: F. Nolan DATE: June 15, 1967
DOCUMENTATION AUTHOR: F. Nolan DATE: June 20, 1973
SOURCE LANGUAGE: FORTRAN IV
APPLICABLE COMPUTERS: IBM Systems 360, 370; CDC 6000 series
REVISION: DATE:

PURPOSE: To provide a convenient conversion between two common arrangements for the storage of triangular (and symmetric) matrices.

ANALYTIC DESCRIPTION: The routine makes systematic use of transpositions, i.e., interchanges of two array elements. It is a well known result in permutation theory that every permutation can be represented as a product of transpositions.

PROGRAM DESCRIPTION: There is no loss of generality in assuming that the input matrix is of lower triangular form. It is natural to store such matrices by row or by column. Both arrangements are illustrated for a matrix of order 5. The understanding is that the (4,3) element, for example, is assigned position 9 using row storage, and position 11 using column storage.

Row Storage					Column Storage				
1					1				
2	3				2	6			
4	5	6			3	7	10		
7	8	9	10		4	8	11	13	
11	12	13	14	15	5	9	12	14	15

Given a lower triangular or symmetric matrix, stored in either fashion, SWITCH can re-arrange it to the other form. The re-arrangement is carried out "in place" in the sense that no auxiliary array is required. For an input matrix of order m, the transformation is performed in approximately $\frac{1}{4}m^2$ transpositions. There are no rounding errors.

PROGRAM RESTRICTIONS: The matrix must be of order at least 3.

INPUT PARAMETERS:

FORTRAN Name	Description
A	Singly-dimensioned real array containing the matrix to be re-arranged.
M	Order of matrix A. If M is given positive, conversion is from row to column storage. If M is given negative, conversion is from column to row storage.

OUTPUT PARAMETERS:

A	Matrix in re-arranged order.

CALLING SEQUENCE:

```
CALL SWITCH (A, M)
```

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APPENDIX E

DISCUSSION OF NONLINEARITIES AND TIME DEPENDENCY OF h AND T_{AW}*

The heat conduction problem that CAVE solves generally contains nonlinearities as a consequence of the material properties being temperature dependent and the radiation heat transfer taking place at the surface of the structure. The problem usually has the further complication of time varying boundary conditions by virtue of h and T_{AW} being time dependent. This appendix considers two questions: (1) how should the nonlinearities be handled; and (2) how should the time dependence of h and T_{AW} be handled.

Considering the second question first, it is noted that the time dependence does not destroy the linearity of the problem. However, in this case an eigenvalue solution cannot be obtained by treating h(t) directly in the system of ordinary differential equations. This is seen by considering the restrictive case in which

$$h(y,t) = h_o(y) + h_1(t)$$

If the semidiscrete problem has the following form (Refer to eq. A-1a)

$$\frac{d\underline{T}}{dt} = A(t)\,\underline{T} + \underline{b}(t); \qquad \underline{T}(0) = \underline{T}_o$$

where for simplicity M is taken to be the identity matrix, then the diagonal elements of A depend on t and the above assumption on the form of h(y,t) leads to

$$A(t) = A_o + a_1(t)J$$

where $a_1(t)$ is a scalar time function and J is a diagonal matrix with ones and zeros. In this case

$$A(t_1)\,A(t_2) \neq A(t_2)\,A(t_1)$$

which therefore precludes an eigenvalue/eigenfunction solution.

---

*This appendix courtesy of Michael J. Rossi.

Therefore, since the eigenvalue solution does not exist for even this restrictive form on h(y,t), the recommendation then is to subdivide the time interval and take h and T_{AW} to be piecewise constant within each subinterval. Returning to the first of the two questions posed, we see another motive for subdividing the time interval. If the subdivision for h and T_{AW} produces short enough intervals one may account for the temperature-dependent properties and radiation terms by taking them to be piecewise constant depending on the temperature at the beginning of each subinterval. One must caution, however, against taking time subintervals which are so short as to require too large a number of eigenvalues and eigenfunctions in order to ensure an accurate solution of the resulting subproblems. The best approach might be to first predict the temperature history on a subinterval based on a time-invariant linear model, and then to correct the solution by considering a forcing term to account for the total neglected effects of radiation, temperature dependent thermal properties, and time-dependent convection.

APPENDIX F

DERIVATION OF SOLUTION TO THE EQUATION $M\dot{\underline{T}} = B\underline{T} + F$

This appendix presents a step-by-step solution to the vector equation

$$M\dot{\underline{T}} = B\underline{T} + \underline{F} \qquad \text{Eq. (F-1)}$$

which represents the temperature response within a body that has been discretized into a number of uniform temperature systems or nodes that are coupled and changing in temperature. The elements of the diagonal matrix M represent the mass-specific heat product of each system or node; the elements of the matrix B represent the convective-conductive couplings between nodes; and the elements of the vector $\underline{F}$ represent the product of the convective couplings with the corresponding fluid adiabatic wall temperature (or recovery temperature). And, of course, the elements of the vector $\underline{T}$ represent the instantaneous temperatures of the nodes; the elements of the vector $\dot{\underline{T}}$ the time rate of change of the temperatures. Specifically, for the four node system shown in Figure F-1:

$$M = \begin{bmatrix} (\rho VC_p)_1 & 0 & 0 & 0 \\ 0 & (\rho VC_p)_2 & 0 & 0 \\ 0 & 0 & (\rho VC_p)_3 & 0 \\ 0 & 0 & 0 & (\rho VC_p)_4 \end{bmatrix}$$

$$\dot{\underline{T}} = \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \end{Bmatrix}$$

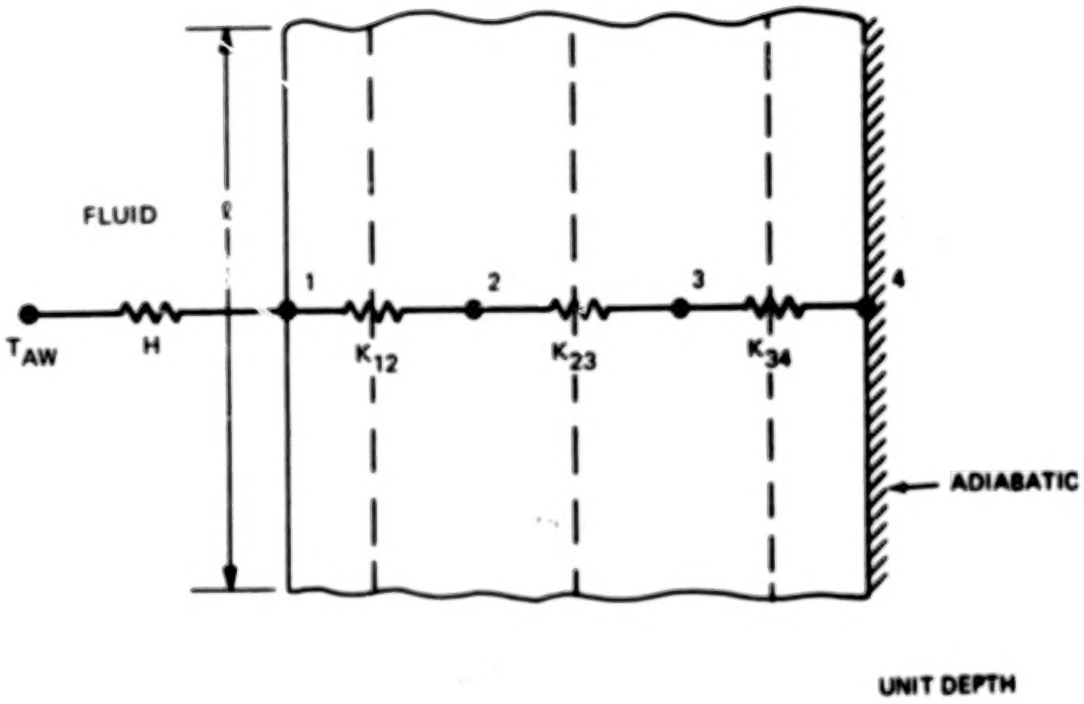

FIG. F-1 FOUR NODE SYSTEM

$$B = \begin{bmatrix} -(H+K_{12}) & K_{12} & 0 & 0 \\ K_{12} & -(K_{12}+K_{23}) & K_{23} & 0 \\ 0 & K_{23} & -(K_{23}+K_{34}) & K_{34} \\ 0 & 0 & K_{34} & -K_{34} \end{bmatrix}$$

$$\underline{T} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix}$$

and

$$\underline{F} = \begin{Bmatrix} H\,T_{AW} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

where,

- ρ = mass density
- V = volume associated with node
- C_p = specific heat
- H = convective coupling, hA
- h = convective heat transfer coefficient
- A = heat transfer area
- K_{ij} = conductive coupling between nodes i and j, $kA/\Delta X_{ij}$
- k = thermal conductivity
- ΔX_{ij} = conduction distance between nodes i and j
- T_{AW} = adiabatic wall temperature of fluid

To obtain the solution to the Eq. (F-1), we premultiply both sides of (F-1) by the inverse of M and obtain

$$\dot{\underline{T}} = M^{-1}B\underline{T} + M^{-1}F \qquad \text{Eq. (F-2)}$$

If we define a symmetric matrix A by

$$A = M^{-1/2}BM^{-1/2} \qquad \text{Eq. (F-3)}$$

we obtain after premultiplication of Eq. (F-3) by $M^{-1/2}$ and postmultiplication by $M^{1/2}$

$$M^{-1/2}\,AM^{1/2} = M^{-1}B \qquad \text{Eq. (F-4)}$$

After substituting Eq. (F-4) 'nto Eq. (F-2) and defini.g a matrix N for convenience in writing by

$$N = M^{-1/2}AM^{1/2} \qquad \text{Eq. (F-5)}$$

we get

$$\dot{\underline{T}} = N\underline{T} + M^{-1}\underline{F} \qquad \text{Eq. (F-6)}$$

We now assume that the solution to Eq. (F-1) can be expressed as

$$\underline{T} = \Theta\ (t) + \underline{T}_{\infty} \qquad \text{Eq. (F-7)}$$

where Θ is a vector having time-dependent components and $\underline{T}_{\infty}$ is a vector that is independent of time. (Physically, $\underline{T}_{\infty}$ represents the steady-state temperatures that the system will achieve.)

Substitution of Eq. (F-7) into Eq. (F-6) yields:

$$\dot{\Theta} = N\Theta + N\underline{T}_{\infty} + M^{-1}\underline{F} \qquad \text{Eq. (F-8)}$$

and we shall let $\underline{T}_\infty$ be such that it satisfies

$$N\underline{T}_\infty + M^{-1}\underline{F} = 0 \qquad \text{Eq. (F-9)}$$

Therefore, we have

$$\underline{T}_\infty = -M^{-1/2}A^{-1}M^{-1/2}\underline{F} \qquad \text{Eq. (F-10)}$$

Then with $\underline{T}_\infty$ satisfying Eq. (F-9), Eq. (F-8) becomes

$$\dot{\underline{\theta}} = N\underline{\theta}$$

which has the solution*

$$\underline{\theta} = e^{tN}\underline{\theta}_0 \qquad \text{Eq. (F-11)}$$

Returning to Eq. (F-7), we have after substituting Eq. (F-11)

$$\underline{T} = e^{tN}\underline{\theta}_0 + \underline{T}_\infty \qquad \text{Eq. (F-12)}$$

To evaluate $\underline{\theta}_0$ we use the initial condition

$$\underline{T}(0) = \underline{T}_i$$

thus

$$\underline{\theta}_0 = \underline{T}_i - \underline{T}_\infty$$

which gives upon substitution into eq. (F-12)

$$\underline{T} = e^{tN}\left[\underline{T}_i - \underline{T}_\infty\right] + \underline{T}_\infty \qquad \text{Eq. (F-13)}$$

*See for example Hochstadt, "Differential Equations," Holt, Rinehart and Winston, pp 55-58.

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Eq. (F-13) is the solution to Eq. (F-1) along with the initial condition; however, from a computational standpoint Eq. (F-13) is not convenient because evaluation of the exponential term requires summing an infinite power series in the matrix N.

We now develop a more convenient form for Eq. (F-13). Recalling the definition of N from Eq. (F-5)

$$N = M^{-1/2} AM^{1/2}$$

we see that

$$N^2 = (M^{-1/2}AM^{1/2})\,(M^{-1/2}AM^{1/2}) = M^{-1/2}A^2M^{1/2}$$

and, in general, that

$$N^j = M^{-1/2}A^jM^{1/2}$$

Therefore, the Taylor series

$$e^{tN} = \sum_{j=o}^{\infty} \frac{t^jN^j}{j!}$$

can be written

$$e^{tN} = M^{-1/2} \sum_{j=0}^{\infty} \frac{t^jA^j}{j!}\ M^{1/2}$$

which leads to the result

$$e^{tN} = e^{tM^{-1/2}\,AM^{1/2}} = M^{-1/2}\,e^{tA}\,M^{1/2} \qquad \text{Eq. (F-14)}$$

Because the matrix A is a symmetric matrix we can find a matrix V such that

$$V^{-1}\,AV = \Lambda \qquad \text{Eq. (F-15)}$$

where Λ is a diagonal matrix formed with the eigenvalues of A.* The matrix V can be selected to be orthogonal, that is, it is a matrix whose columns comprise the elements of n linearly independent eigenvectors of matrix A that are mutually orthogonal and of unit length.

From Eq. (F-15) we have

$$A = V \Lambda V^{-1}$$

which when substituted into Eq. (F-14) yields

$$e^{tN} = M^{-1/2} V e^{t\Lambda} V^{-1} M^{1/2} \qquad \text{Eq. (F-16)}$$

where we have made use of the identity

$$e^{tV\Lambda V^{-1}} = Ve^{t\Lambda} V^{-1}$$

The representation for e^{tN} given in Eq. (F-16) is convenient since Λ is a diagonal matrix and therefore the exponential term is easily and explicitly evaluated as follows:

$$e^{t\Lambda} = \begin{bmatrix} e^{t\lambda_1} & & & 0 \\ & e^{t\lambda_2} & & \\ & & \ddots & \\ 0 & & & e^{t\lambda_n} \end{bmatrix}$$

where the λ's are the eigenvalues of matrix A.

*cf. Hildebrand, "Methods of Applied Mathematics," Prentice-hall, pp. 37-39.

Substituting Eq. (F-16) into Eq. (F-13) we obtain the final form for the solution to equation (F-1)

$$\underline{T} = M^{-1/2} V e^{tA} V^T M^{1/2} \left[\underline{T}_i - \underline{T}_\infty\right] + \underline{T}_\infty \qquad \text{Eq. (F-17)}$$

where we have made use of the property that for normalized modal matrices, the transpose of the matrix equals the inverse of the matrix, i.e.,

$$V^T = V^{-1}$$

Equation (F-17) is Eq. (9) of NASA CR-2435 by Maise and Rossi; it forms the basis of the HAN method.

It should be noted that for a system with n nodes or elements the matrix V will have n columns of eigenvectors and the matrix A will have n eigenvalues along the diagonal. As noted in NASA CR-2435, good approximate solutions for $\underline{T}$ are obtained by finding and using only the "dominant" eigenvectors and eigenvalues of A. Large savings in computer time (factor of ten or more) are achieved by finding and using only say 5 dominant eigenvectors and eigenvalues for a system of 100 nodes or more.

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16. Abstract

This report describes a digital computer code CAVE (Conduction Analysis Via Eigenvalues), which finds application in the analysis of two-dimensional transient heating of hypersonic vehicles. The code is an extension of the work reported in NASA CR-2435 for the inverse conduction problem. CAVE is written in FORTRAN IV and is operational on both IBM 360-67 and CDC 6600 computers.

The method of solution is a hybrid analytical-numerical technique that is inherently stable permitting large time steps even with the best of conductors having the finest of mesh size. This method can provide a factor-of-five reduction in machine time compared to conventional explicit finite difference methods when structures with small time constants are analyzed over long flight trajectories. The aerodynamic heating boundary conditions are calculated by the code based on the input flight trajectory (i.e., altitude, velocity and angle of attack as functions of time) or can optionally be calculated external to the code and then entered as input data. The code computes the network conduction and convection links, as well as capacitance values, given basic geometrical and mesh sizes, for four generations (leading edges, cooled panels, X-24C structure and slabs) or optionally there values can be input directly.

This report is primarily a user's manual for the CAVE code. Input and output formats are presented and explained. Sample problems are included. A brief summary of the hybrid analytical-numerical technique, which utilizes eigenvalues (thermal frequencies) and eigenvectors (thermal mode vectors) is given in an appendix. Other appendixes include the aerodynamic heating equations that have been incorporated in the code and flow charts.

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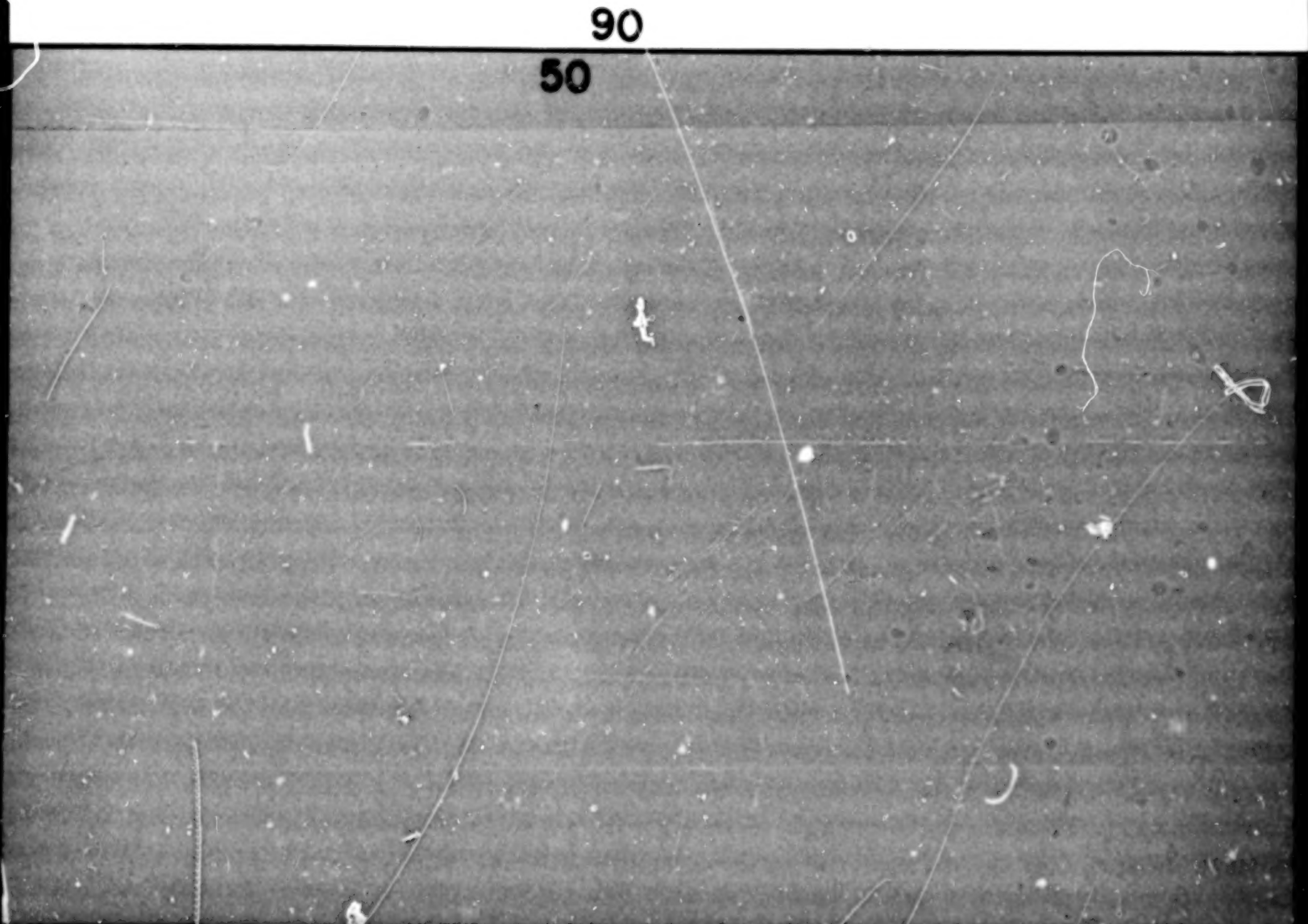
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